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THE THOMSON EFFECTS IN TUNGSTEN, TANTALUM AND CARBON AT INCANDESCENT TEMPERATURES DETERMINED BY AN OPTICAL PYROMETER METHOD.

By A. G. Worthing.

Introduction.

THE present paper is a direct outcome of certain effects noted in connection with a study by Hyde, Cady and Worthing[1] of the energy losses in electric incandescent lamps. Favorable conditions for observing the Thomson effect exist in such lamps in consequence of the high temperature gradients in the neighborhood of the leading-in junctions and of the large current densities. W. König,[2] using a visual method, has obtained qualitative results on Pt, Cu, Fe and constanstan. However, so far as the writer knows, no quantitative results have been obtained hitherto in connection with incandescent temperatures. An optical pyrometer method for such measurements and its application are here described.

Theory.

Consider a small filament of uniform surface and cross-section mounted in an evacuated bulb and heated to incandescence by an electric current. For the temperature distributions in the neighborhood of a cooling junction for a given intensity of heating current, three cases are to be distinguished depending on whether the heating current through the filament (1) is a direct current directed away from the junction, (2) is a direct current directed toward the junction, or (3) is an alternating current. In Fig. 1 (a diagrammatic representation for tungsten) the curves A_1, A_2 and A_3 respectively represent the distributions of the rates of production of heat per unit of radiating surface, w_1, w_2 and w_3 (Table I.); and the curves B_1, B_2 and B_3 respectively the radiation intensities E_1, E_2 and

[1] Illum. Eng. (Lond.), *4*, p. 389, 1911. Trans. Illum. Eng. Soc. (U. S.), *6*, p. 238, 1911.
[2] Phys. Zeit., *11*, 1913, 1910.

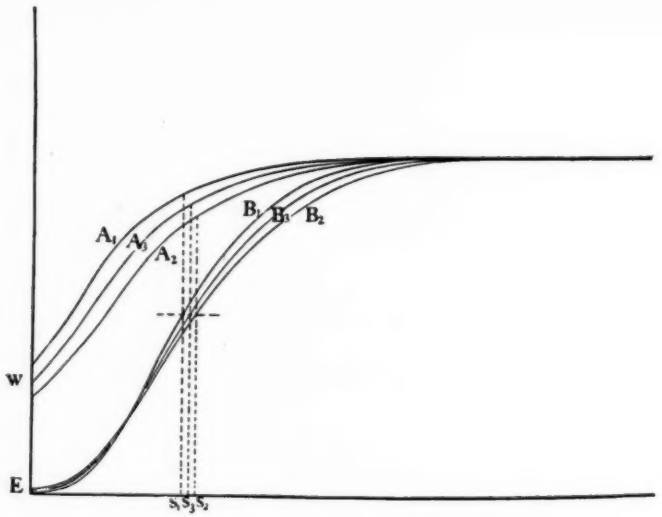

Fig. 1.

Diagrammatic representation for a portion of a tungsten filament near a cooling junction, of the distributions of the rates of development of heat in the elements of filament per unit of radiating surface (A_1, A_2, A_3) and of the radiation intensities (B_1, B_2, B_3) respectively for the conditions that the heating current is (1) directed away from the cooling junction, (2) directed toward the cooling junction, and (3) alternating.

TABLE I.

Symbols and Quantities Used.

w = Rate of development of heat in an element of filament per unit of radiating surface.
E = Radiation intensity.
l = Distance along filament.
r = Radius of filament.
I = Current.
ρ = Resistivity.
T = Temperature.
σ = Coefficient of the Thomson effect.
$\frac{dH}{dT}$ = Rate of heat conduction along the filament at a given cross-section.
k = Thermal conductivity.
p = Constant defined by equation (8).
μ = Constant of equation (12).
E_m and T_m = Maximum values of E and T.

E_3. Methods for obtaining curves B_1, B_2, B_3 and A_3 are fully described by the author[1] elsewhere. A method which may be employed in obtaining curves A_1 and A_2 will appear from later considerations. The crossing of the B curves has no particular significance from the standpoint

[1] PHYS. REV., II., 4, p. 535, 1914.

of the Thomson effect. It is merely a consequence of the lateral displacements of these curves due to the Peltier effect at the junction.

For the rates w_1 and w_2 corresponding to the condition $E_1 = E_2$ and thus necessarily $T_1 = T_2$ (positions S_1 and S_2, Fig. 1), we have, from the standpoint that the heating per unit length of filament must equal the sum of the Joulean and the Thomson heatings,

$$2\pi r w_1 = \frac{I^2\rho}{\pi r^2} - \sigma I \frac{dT}{dl_1}, \tag{1}$$

and

$$2\pi r w_2 = \frac{I^2\rho}{\pi r^2} + \sigma I \frac{dT}{dl_2}, \tag{2}$$

in which the sign of the coefficient σ is taken as positive when the gradient of the Thomson E.M.F. coincides in direction with the temperature gradient. For tungsten as is shown in Fig. 1, σ is negative. (1) and (2) give directly

$$\left[\sigma = \frac{2\pi r(w_1 - w_2)}{I\left(\frac{dT}{dl_1} + \frac{dT}{dl_2}\right)}\right]_{T_1=T_2=T}. \tag{3}$$

In order to make use of (3), a method of obtaining the distributions of w_1 and w_2 must be devised. Since the rate of heat production in an element of filament must equal the sum of the rate of radiation of energy from it and of the *net* rate of heat conduction from it, there results

$$2\pi r w = 2\pi r E - \frac{d}{dl}\left(\frac{dH}{dt}\right), \tag{4}$$

where dH/dt represents the rate of conduction of heat across the cross-section of filament at l. Since

$$\frac{dH}{dt} = \pi r^2 k \frac{dT}{dl}, \tag{5}$$

there follows

$$\frac{d}{dl}\left(\frac{dH}{dt}\right) = \pi r^2\left[k\frac{d^2T}{dl^2} + \frac{dk}{dT}\left(\frac{dT}{dl}\right)^2\right]. \tag{6}$$

Equations (4) and (6) give

$$w = E - \frac{r}{2}\left[k\frac{d^2T}{dl^2} + \frac{dk}{dT}\left(\frac{dT}{dl}\right)^2\right]. \tag{7}$$

Determinations of k as a function of T and of T and E as functions of l, such as the writer has reported in the papers already referred to, enable one to obtain the desired distributions of w_1 and w_2 as functions first of E_1 and E_2 and then of l. The relations thus obtained suffice for obtaining σ.

A simplification results if a certain condition as to the distributions of E_1 and E_2 exists. This condition, which was found experimentally to hold, is

$$\left[\frac{dT}{dl_2} = p\frac{dT}{dl_1}\right]_{T_1=T_2=T}, \tag{8}$$

in which p is a constant depending on the material of the filament and on the maximum filament temperature. Then since the distribution of E_3 is approximately if not accurately a mean of the distributions of E_1 and E_2, this condition may be rewritten as

$$\left[\sqrt{p}\frac{dT}{dl_1} = \frac{1}{\sqrt{p}}\frac{dT}{dl_2} = \frac{dT}{dl_3}\right]_{T_1=T_2=T_3=T}. \tag{9}$$

Since as indicated in (8) and (9) the points considered on the different temperature distribution curves refer to cross-sections having the same temperature, l_1 and l_2 may be expressed as functions of l_3. This leads to

$$\left[p\frac{d^2T}{dl_1^2} = \frac{1}{p}\frac{d^2T}{dl_2^2} = \frac{d^2T}{dl_3^2}\right]_{T_1=T_2=T_3=T}. \tag{10}$$

Equations (3) and (7) when combined subject to (9) and (10) give finally

$$\sigma = \frac{\pi r^2}{2I}\left(p - \frac{1}{p}\right)^k \frac{\dfrac{d^2T}{dl_3^2} + \dfrac{dk}{dT}\left(\dfrac{dT}{dl_3}\right)^2}{\dfrac{dT}{dl_3}}. \tag{11}$$

RESULTS.

As indicated in the paper already referred to, the temperature distribution along an incandescent tungsten filament in vacuo is of the type

$$\frac{T}{T_m} = [1 - e^{-\mu(l+l_0)}]^{1.87}. \tag{12}$$

Such a temperature distribution curve and the corresponding radiation intensity distribution curve were platted for a particular tungsten filament (r = 0.01045 cm.) heated by a current of 4.32 amps. to a maximum temperature of 2315° K. (scale of Mendenhall and Forsythe).[1] In Fig. 2 there are platted the two individual distributions of radiation intensity depending on the direction of flow of the D.C. heating currents, from which the previously mentioned curves were obtained. As platted they are approximately straight lines. As has been stated before in connection with the other related curves, the deviations, with the exception of those near the cooling junction, are accidental. The conclusion as to the constancy of p resulted from a large number of such tests. In Table II.

[1] Astrophys. Jour., 37, p. 380, 1913.

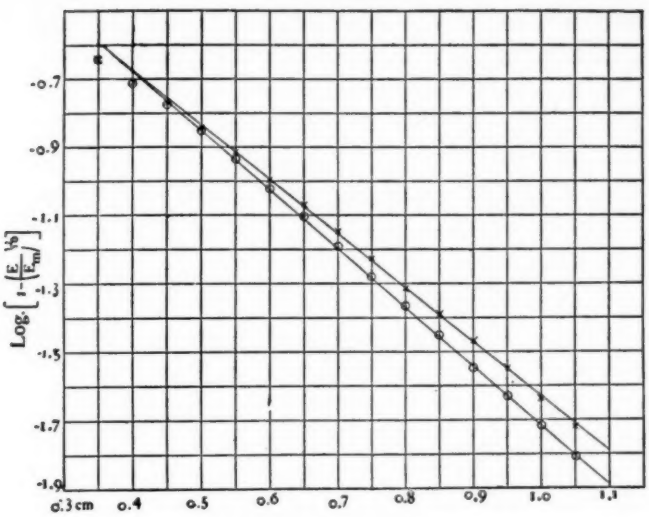

Fig. 2.

The radiation intensity distribution for a particular tungsten filament ($r = 0.01045$ cm.) heated to a maximum temperature of 2315° K., by (1) a current of 4.32 amps. directed away from the cooling junction (⊙ ⊙ ⊙), and (2) by a current of the same intensity directed toward the cooling junction (× × ×).

TABLE II.

Data Necessary for the Application of (11) to a Particular Tungsten Filament, and Computed Values of the Thomson E.M.F. Coefficient.

Condition.	Maximum Filament Temperature.	Average Value of p.	r	I	μ
1	2315° K.	0.920	0.01045 cm.	4.32 amp.	$3.80 \frac{1}{\text{cm.}}$
2	1890	0.955	0.01045	2.88	2.65

Condition 1.		Condition 2.	
Temperature in ° K.	σ in $\frac{\text{Microvolts}}{\text{Degree}}$.	Temperature in ° K.	σ in $\frac{\text{Microvolts}}{\text{Degree}}$.
2200	−35	1800	−16
2000	−28	1600	−11
1800	−21	1500	−10

there are given the average values of p for two maximum filament temperatures. The value for 1890° K. is considerably more uncertain than that for 2315° K. There are given also other data necessary in the application of these two values by means of (11), together with the results. The lower temperature in each case refers to a region in which the

fit of (12) indicates the application of (11) as being still justifiable. The lack of agreement of the two values for 1800° K., on the assumption that σ is independent of the current density, is probably ascribable to the comparatively great difficulty experienced in condition 2 in determining the value of p. That thermionic currents can not account for the difference seems evident when one considers the maximum potential difference of approximately 20 volts between the terminals of the lamp filament in conjunction with the work of Langmuir.[1]

The results obtained for filaments of tantalum and of untreated carbon

TABLE III.

Thomson Effect in Tantalum and Carbon.

Tantalum.		Carbon.	
Temperature in ° K.	σ in $\frac{\text{Microvolts}}{\text{Degrees}}$.	Temperature in ° K.	σ in $\frac{\text{Microvolts}}{\text{Degree}}$.
2100	+24	2100	−22
1900	+20	2000	−21
1700	+16	1800	−19

are indicated in Table III. The temperature calibrations here used are also those of Mendenhall and Forsythe.[2]

The greatest uncertainty in this work lies in the fact that the relations $E = \varphi(T)$ and $E = f(l)$ [relations (5) and (6) of previous paper on thermal conduction] are determined at different portions of the filament under experiment. This means a difference in the two cases in the glassware between the background filament and the pyrometer filament and also possible inequalities in the cross-sections of the filament. Due to the many filaments of tungsten which have been investigated, the uncertainty there has been largely reduced but such is not the case with carbon or tantalum. It is possible to eliminate largely this source of error by using metal cases with plate glass windows, with a device for clamping and unclamping the filament in the neighborhood to be studied.

SUMMARY.

1. A method of studying quantitatively the Thomson effects in filaments mounted in evacuated chambers, with the aid of an optical pyrometer, has been developed.

2. The Thomson E.M.F. coefficients have been determined for tungsten, tantalum and carbon (Tables II. and III.) for temperatures ranging from 1500° K. to 2200° K.

[1] PHYS. REV., II., 2, p. 450, 1913.

[2] Astrophys. Jour., 37, p. 380, 1913.

The writer is indebted to Messrs. K. Linder, G. Cadisch, A. Shapero and A. Scheel for efficient aid in the experimental work, and particularly to Dr. E. P. Hyde, for first suggesting the quantitative determinations here recorded.

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MUTUAL INDUCTANCES OF CIRCUITS COMPOSED OF STRAIGHT WIRES.

BY GEORGE A. CAMPBELL.

THE mutual inductance between any two circuits made up of m and n straight wires of negligible diameter may be most simply expressed as the sum of the mn mutual Neumann integrals between the sides taken in pairs, one from each circuit. As such inductances are required in practical computations, it is desirable to have a formula for the mutual Neumann integral between two skew lines of any lengths in any relative location. The Bureau of Standard's collection of "Formulas and Tables for the Calculation of Mutual and Self-Induction (Revised)" does not contain such a formula and the only statement of the result[1] which I have seen is involved and unsatisfactory for actual use. A general formula in convenient form, formulas for a number of special cases, a diagram for use in calculations, and, finally, the deduction of the formulas follow.

To speak of the self and mutual inductances of circuits one or both of which are unclosed, is logically inexact and practically unsafe, for it tends to vague thinking and the entire neglect of the return circuit in cases where the effect of the return is easily lost sight of because it is at a remote distance. Heaviside advocated the exclusive consideration of closed circuits, securing external continuity in every case by means of two superposed uniform radial systems, one diverging from the positive terminal in all directions, the other converging on the negative terminal from all directions. This means subtracting one half of the second order difference $(-Aa + Ab + Ba - Bb)$ of the distances between the terminals $A, B; a, b$, from the Neumann integral for any unclosed circuits between these terminals. A better way, it seems to me, is to continue using the Neumann integrals for unclosed circuits but to refer to them as the self or mutual Neumann integrals according as the two unclosed circuits are or are not identical. This reserves the terms self and mutual inductances for use with closed circuits exclusively. The following results are expressed in this way.

[1] Martens, F. F., Ann. der Phys., *29*, p. 959, 1909.

Neumann Integral for Skew Lines.

The mutual Neumann integral between any two straight filaments AB, ab, the positive directions A to B and a to b making with each other the angle e ($0 \gtrless e \gtrless \pi$), *is*

$$\left.\begin{aligned} N = \underline{pB'} \log \frac{Bb + \underline{B'b}}{Ba + \underline{B'a}} - \underline{pA'} \log \frac{Ab + \underline{A'b}}{Aa + \underline{A'a}} + \underline{Pb'} \log \frac{bB + \underline{b'B}}{bA + \underline{b'A}} \\ - \underline{Pa'} \log \frac{aB + \underline{a'B}}{aA + \underline{a'A}} - \frac{Pp\Omega}{\tan e} \\ = 2\underline{pB'} \tanh^{-1} \frac{ab}{aB + Bb} - 2\underline{pA'} \tanh^{-1} \frac{ab}{aA + Ab} \\ + 2\underline{Pb'} \tanh^{-1} \frac{AB}{Ab + bB} - 2\underline{Pa'} \tanh^{-1} \frac{AB}{Aa + aB} - \frac{Pp\Omega}{\tan e} \end{aligned}\right\} \quad (1)$$

where A'B', a'b' are the projections of AB and ab on each other; Pp is the common perpendicular to AB, ab taken positive, as are all other distances except those (underscored) measured along AB and ab which are taken algebraically positive in the directions AB and ab respectively; Ω is the (positive) solid angle subtended at B by a parallelogram abcd constructed on ab with bc parallel and equal to AB.[1]

Special Cases.

1. Filaments mutually perpendicular, $N = 0$.

2. Filaments starting from a common point ($P = A = p = a$),

$$N = \underline{A'B'} \log \frac{Bb + \underline{B'b}}{AB + \underline{B'a}} + \underline{a'b'} \log \frac{bB + \underline{b'B}}{ab + \underline{b'A}}. \quad (2)$$

3. Filaments mutually parallel ($e = 0$ or π),

$$N = AB \log \frac{aB + \underline{a'B}}{bB + \underline{b'B}} + \underline{Ab'} \log \frac{bB + \underline{b'B}}{bA + \underline{b'A}} - \underline{Aa'} \log \frac{aB + \underline{a'B}}{aA + \underline{a'A}} - (-Aa + Ab + Ba - Bb) \quad (3)$$

3*a*. If $e = 0$, and the midpoints of AB and ab are opposite each other.

$$N = AB \log \frac{2Ab + AB + ab}{2Aa + AB - ab} + ab \log \frac{2Ab + AB + ab}{2Aa + ab - AB} + 2Aa - 2Ab, \quad (3a)$$

4. Filaments mutually parallel beginning at a common perpendicular, with positive direction either the same or opposite ($e = 0$ or π, $Aa = Pp$),

$$N = AB \log \frac{aB + AB}{bB + \underline{b'B}} + ab \log \frac{Ab + ab}{Bb + \underline{B'b}} - (-Aa + Ab + Ba - Bb) \quad (4)$$

[1] Formula 10 may be used for Ω.

$$\left.\begin{aligned} &= 2AB \log \frac{aB + AB}{Aa} - 2(Ab - Aa) \\ &= 4AB \tanh^{-1} \frac{AB}{aB + Bb} - 2Ab + 2Aa \\ &= 2AB \left(\log \frac{2}{t} - 1 + t - \frac{t^2}{4} + \frac{t^4}{32} - \frac{t^6}{96} \right. \\ &\qquad \left. + \frac{5t^8}{32.32} - \cdots \right), \quad t = \frac{Aa}{AB} \end{aligned}\right\} \text{ if } AB = ab,\ e = 0 \quad (5)$$

$$\left.\begin{aligned} &= 2AB \log \frac{aB + AB}{bB + 2AB} - (-Aa + 2Ab - Bb) \\ &= 2AB \left(-\log 2 + \frac{t}{2} - \frac{3t^2}{16} + \frac{15t^4}{16.32} - \frac{63t^6}{64.96} \right. \\ &\qquad \left. + \frac{5.255t^8}{16.16.32.32} - \cdots \right), \quad t = \frac{Aa}{AB} \end{aligned}\right\} \text{ if } AB = ab,\ e = \pi. \quad (6)$$

5. Non-overlapping portions of a straight filament $ABab$ ($e = 0$, $Pp = 0$)

$$\begin{aligned} N &= AB \log \frac{Ab}{Aa} + ab \log \frac{Ab}{Bb} + Ba \log \frac{Ab \cdot Ba}{Aa \cdot Bb} \\ &= -Aa \log Aa + Ab \log Ab + Ba \log Ba - Bb \log Bb. \end{aligned} \quad (7)$$

6. Filament ab with the element at the point s having the algebraic projection dx on filament AB.

$$\begin{aligned} dN &= \log \frac{sB + s'B}{sA + s'A} dx \\ &= 2dx \tanh^{-1} \frac{AB}{As + sB}, \end{aligned} \quad (8)$$

RULE FOR USING THE DIAGRAM OF CONFOCAL ELLIPSES (FIG. 1) FOR FINDING THE MUTUAL NEUMANN INTEGRAL BETWEEN A FINITE STRAIGHT LINE AB AND ANY OTHER LINE ab.

Draw the two lines on such a scale that A, B coincide with the foci; if AB and ab do not both lie in one plane bring ab into the plane of the paper by rotating it point by point about the right line through AB. Consider the ellipses to be contour lines with the elevations noted upon them. Determine the projection upon the vertical plane through the foci A, B of the vertical cylindrical surface erected upon ab and bounded by the contour surface, considering areas to be positive or negative according as the projection of ab has or has not

the same positive direction as AB. If horizontal distances have been measured in 10^9 cm. the area found is the mutual integral in henries.

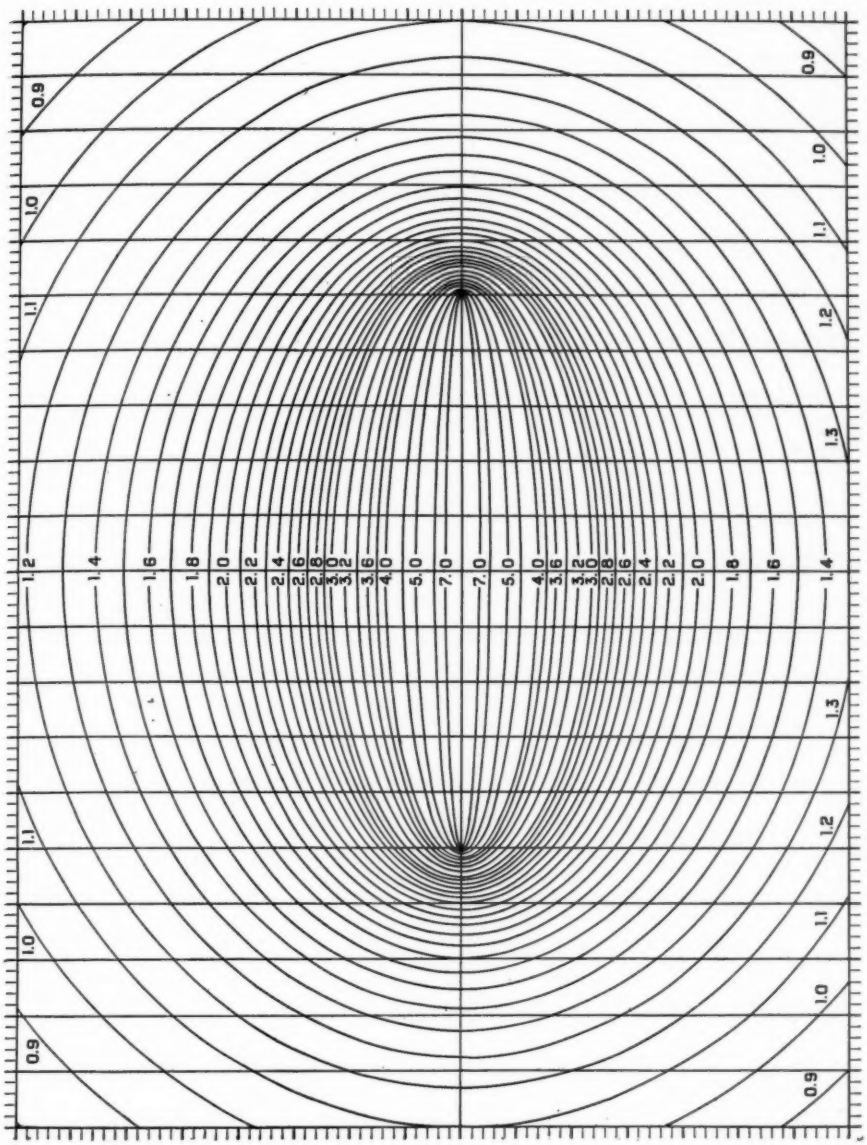

Fig. 1.

Confocal Ellipse Diagram for Calculating Mutual Neumann Integrals.

TRANSFORMATIONS OF FORMULA 1.

Formula (1) may be thrown into a variety of equivalent forms by means of the following geometrical relationships for the first ratio oc-

curring in the logarithmic terms, which is typical of all, and for the solid angle, the expression for which is in terms of the dihedral angles of the tetrahedron on AB, ab.

$$\frac{Bb + \underline{B'b}}{Ba + \underline{B'a}} = \frac{Ba - \underline{B'a}}{Bb - \underline{B'b}} = \frac{(Bb + \underline{B'b})(Ba - \underline{B'a})}{B'B^2}$$
$$= \cot\frac{\angle Bab}{2}\cot\frac{\angle Bba}{2} = \frac{Ba + Bb + ab}{Ba + Bb - ab} = e^{2\tanh^{-1}\frac{ab}{aB+Bb}}, \tag{9}$$

$$\Omega = \tan^{-1}\left(\frac{Pp}{Bb}\cot e + \frac{PB}{Pp}\frac{pb}{Bb}\sin e\right) - \tan^{-1}\left(\frac{Pp}{Ba}\cot e + \frac{PB}{Pp}\frac{pa}{Ba}\sin e\right)$$
$$- \tan^{-1}\left(\frac{Pp}{Ab}\cot e + \frac{PA}{Pp}\frac{pb}{Ab}\sin e\right) + \tan^{-1}\left(\frac{Pp}{Aa}\cot e + \frac{PA}{Pp}\frac{pa}{Aa}\sin e\right). \tag{10}$$

PROOFS.

The Neumann integral is

$$N = \cos e \int\int \frac{dSds}{r}, \quad \text{where } r^2 = Pp^2 + S^2 - 2Ss\cos e + s^2$$

if S, s are measured in the positive directions along AB, ab respectively from the common perpendicular Pp. As is easily shown by performing the indicated differentiations, the integral for N may be written in the following directly integrable form.

$$N = \cos e \int\int \left(D_S\frac{S}{r} + D_s\frac{s}{r} - \frac{Pp^2}{r^3}\right) dSds$$
$$= \cos e \left(\left|S\int\frac{ds}{r}\right| + \left|s\int\frac{dS}{r}\right| - \frac{Pp}{\sin e}\int\int\frac{Pp}{r^3}\sin e\, dSds\right)$$
$$= \cos e \,||\, S\log(r + s - S\cos e) + s\log(r + S - s\cos e)$$
$$- \frac{Pp\ (\text{solid angle})}{\sin e}\Bigg|_{\substack{S=\underline{PB} \\ S=\underline{PA}}}\Bigg|_{s=\underline{pb}}^{s=\underline{pa}}.$$

the solid angle being given by the last integral, as ($\sin e\, dSds$) may be taken as an element of area of the parallelogram $abcd$ in oblique coördinates (if $\sin e$ is positive which requires that e shall lie between 0 and π), the factor Pp/r reduces this to its projection normal to r, and division by r^2 gives the solid angle subtended by the element. Introducing the limits and substituting $\underline{PB}\cos e = \underline{pB'}$, etc., give formula (1).

This Neumann integral is also (1) the Newtonian potential at B of the parallelogram $abcd$ with a uniform mass equal to $\cot e$ per unit area, which

suggests other ways of performing the integration; (2) the mutual potential of two rods AB, ab with the mass $\sqrt{\cos e}$ per unit length; (3) the mutual Neumann integral for any arc of a hyperbola and any part of the conjugate axis as is shown below.

Formula (3) is obtained by taking Pp coincident with AA' and changing the first term so as to substitute algebraical quantities measured along AB in place of the algebraical quantities measured along ab. This change is made by noting that for any pair of points X, y on parallel filaments and their projections X', y' the following relations hold, $\underline{X'y} = \pm \underline{Xy'} = \mp \underline{y'X}$, the upper and lower signs applying to positive directions in the same or opposite sense respectively; whence

$$\underline{A'B'} \log \frac{Bb + \underline{B'b}}{Ba + \underline{B'a}} = \pm AB \log \frac{Bb \mp \underline{b'B}}{Ba \mp \underline{a'B}} = AB \log \frac{aB + \underline{a'B}}{bB + \underline{b'B}}.$$

The solid angle term becomes 0/0 but may readily be evaluated, or it may be derived directly from the indefinite integral for this term, which is found to reduce to $\| r \|$ for parallel filaments as $D_S D_s r = -Pp^2 r^{-3} \cos e$ for this case, and substituting this makes the term directly integrable. As the location of Pp is arbitrary, any one of the four terms in (1) may be made to vanish by locating Pp at A, B, a or b. The logarithmic terms may, if desired, be so combined as to be symmetrical in the quantities involved.

Formula (4) is a special case of (3). Formula (5) is the well-known result for the opposite sides of a rectangle, and formulas (3), (4) and (6) may all be derived from it, in spite of the fact that it is merely a special case of (3) and (4).

Formula (7) holds only for the particular sequence of points indicated, but any non-overlapping sequence may be readily reduced to this. The second expression is the one which is most readily obtained by direct integration of Neumann's integral, for that gives $\| -r \log r \|$ for the indefinite integral in this special case.

Formula (10) (which corresponds to the indefinite integral $\| \tan^{-1} ((Pp^2 \cot e + Ss \sin e)/Ppr) \|$ for the solid angle expressed in terms of the dihedral angles may be checked by differentiating with respect to both S and s, or it may be derived by geometrical considerations.

Formula (8) expresses the mutual Neumann integral between any filament ab and any straight filament AB in terms of the projections of the elements of ab on AB multiplied by factors which depend only upon the location of the elements with respect to AB. A diagram showing the value of this factor at every point may be used for determining the value of the integral for any particular filament ab. To find the locus of the point s for constant values of this factor, it is sufficient to notice that

formula (8) (second form) remains unchanged if the denominator $(As + sB)$ is constant, which gives an ellipse with foci at A and B; the ellipticity $= AB/(As + sB)$ is, by formula (8), equal to $\tanh (dN/2dx)$. A family of confocal ellipses has therefore been drawn for different values of dN/dx, and is reproduced in the accompanying figure. Since rotating the elements of ab about AB does not change formula (8), the diagram may be used for all lines including those which do not lie in the plane of AB by following the directions given above. The accuracy obtainable in this method of calculating Neumann integrals depends only upon the number of ellipses shown on the diagram and the precision of the graphical work.

The rotation of a finite straight skew filament ab point by point about AB into the plane of AB, Pp changes it into an arc of a hyperbola having the common perpendicular Pp for a semi-transverse axis and asymptotes through P making the angle e with AB. For, take any point s on ab and set $ss' = x$, $Ps' = y$, then

$$x^2 = y^2 \tan^2 e + Pp^2$$

or

$$\frac{x^2}{Pp^2} - \frac{y^2}{Pp^2 \cot^2 e} = 1.$$

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THE VARIATION WITH METEOROLOGICAL CONDITIONS OF THE AMOUNT OF RADIUM EMANATION IN THE ATMOSPHERE, IN THE SOIL GAS, AND IN THE AIR EXHALED FROM THE SURFACE OF THE GROUND, AT MANILA.

BY J. R. WRIGHT AND O. F. SMITH.

IN the Philippine Journal of Science of February, 1914, the authors published the results of an extensive series of observations on the amount of radium emanation in the atmosphere of Manila. The well-known charcoal-absorption method was used and the main part of the investigation was preceded by numerous tests on certain points concerned with the accuracy of the method. The work at Manila was supplemented by an investigation of the variation of the emanation content of the atmosphere with altitude, observations being taken on Mt. Pauai, elevation 2,460 meters, and the results compared with those for sea level.[1]

Although the especial points of the investigation were those above mentioned we felt justified at that time, since the period involved extended over almost a year and a half, in pointing out certain evident relations between the emanation content of the atmosphere and the different meteorological factors. The data at our command was not considered sufficient, however, to justify the drawing of more than very general conclusions. During the past year observations have been taken with the object of determining more definitely to what extent the amount of radium emanation in the air is dependent on weather conditions. At the same time tests have been made on several other points which have a more or less direct bearing on the question.

The experimental method used was described in detail in our previous paper, but for the sake of completeness a brief summary of the description will be given, together with such modifications as experience had shown to be advisable.

The method had its origin in the discovery by Rutherford that charcoal made from the shells of coconuts possesses the property of absorbing radioactive emanations. Eve[2] and Satterly[3] independently applied the discovery to the determination of the radium-emanation content of the atmosphere.

[1] Phys. Zeit., *15*, 31, 1914.
[2] Phil. Mag., *14*, 724, 1907.
[3] Phil. Mag., *16*, 584, 1908.

To attain this end air was passed at a known rate and for a definite length of time through a tube containing coconut charcoal, which absorbed the emanation from the air. At the same time air was bubbled through a solution of radium bromide, containing a known amount of radium, and the emanation from the solution and the air collected in another charcoal tube. The emanation absorbed in the charcoal was then driven off by heating, collected over water in aspirators, and finally measured by passing into an ionization chamber connected either with an electroscope or an electrometer. By this method quantitative determinations of the amount of radium emanation in the atmosphere can be obtained, and since the method is a comparative one the results should be independent of variations in the pressure, humidity, and nucleation of the air, which is a serious objection to the extensively used active-deposit method.

Since in the present investigation the object was to determine the variation of the emanation content of the air with weather conditions it was essential that all observations should be taken under identical experimental conditions and in the same manner. Having had two years experience in taking similar observations we were able to attain this end with certainty. Throughout the thirteen months covered by this series of experiments the arrangement of apparatus and the method of procedure in taking observations was never varied. Consequently, the results should at least show quantitatively the variation of the amount of emanation in the atmosphere.

EXPERIMENTAL PROCEDURE.

Collecting.—The air to be tested was drawn in through a tube, projecting from a second story window of the Bureau of Science, by means of a motor driven oil pump. By placing several large bottles, having a total capacity of about 50 liters, in series with the pump a constant rate of flow could be maintained for any desired length of time. The rate of flow was determined by means of accurately calibrated oil manometers across glass capillary tubes of fairly small bore. The air stream was regulated by means of easily adjusted pinch cocks on rubber tubing.

To extract the dust from the air a tube containing cotton wool was placed between the intake and a large distributing bottle. From the distributing bottle the air was divided into two exactly equal parts, one part passing through the branch containing the radium bromide solution and the other part through an exactly similar system, with the exception of that part connected with the standard solution. In both branches the air was thoroughly dried by bubbling it first through sulphuric acid and then passing it over calcium chloride.

The coconut charcoal was contained in electro-silica tubes having a uniform bore of 1.5 cm. In each tube there was placed 70 grams of finely granulated coconut charcoal, which was firmly packed and held in place by long plugs of asbestos wool. Experience had shown the importance of keeping the charcoal firmly and uniformly packed if the same fraction of the total amount of emanation passing through the tubes was to be absorbed in every case. Two of these tubes were placed in series in each branch of the collecting system. Tests, which have been described in detail in an earlier paper, had proven that two such tubes in series absorb practically all the emanation passing through them during a 20-hour run. Each set of tubes was permanently labeled and their relative absorptive powers carefully determined for the conditions under which they were to be used.

The bottle containing the radium bromide solution was so arranged that it could be heated by immersing in a solution of sodium chloride. A large spherical condenser was attached to the bottle to prevent loss of the radium bromide during the process of heating. The standard used throughout the entire series of determinations contained 6.28×10^{-10} grams of radium. Preceding each test the solution was put into the so-called " steady state " by bubbling air through the boiling solution for one hour and then through the cool solution for two hours. Our preliminary work had shown conclusively that bubbling air through the cool solution did not remove all the acculumated emanation, nor remove it as rapidly as formed even after the solution had been put in the " steady state." Under the conditions maintained throughout the tests approximately 80 per cent. of the emanation was removed from the solution. This is absolutely independent of the question of what per cent. of the total amount passing over the charcoal is absorbed. In comparative tests the only point of importance in regard to the absorption is that each branch shall absorb the same fraction of the total amount passing through it.

Testing.—The testing apparatus used was a Spindler and Hoyer aluminum-leaf electroscope with an ionization chamber attached. The aluminum leaf in these electroscopes has a fine quartz fiber attached to one edge which makes it possible to obtain very accurate readings with the aid of the reading microscope. The ionization chamber was 38 cm. high and 7.8 cm. in diameter, giving a volume of 1,820 c.c. It was provided with two outlet tubes so that the chamber could easily be exhausted and refilled with the air containing the emanation. The electroscope with the attached ionization chamber had an electrical capacity of 8.7 e.s. units, the range of the scale of 100 divisions being approximately

from 368 to 302 volts. The potential gradient, therefore, was sufficient to produce saturation currents for the degree of ionization dealt with in the experiments. The natural leak was exceedingly constant and had a value of 0.016 division per minute.

The ionization chamber was permanently attached through one opening to a mercury manometer and through the other opening to a Geryk oil pump and to two aspirator bottles, all connected in parallel, so that any one could be put in direct connection with the chamber. Between the aspirators and ionization chamber were placed two small U tubes, one containing calcium chloride and the other phosphorus pentoxide, permitting all the air passing into the chamber to be thoroughly dried.

After a few preliminary experiments we adopted the following method of taking measurements on the emanation collected. The charcoal tubes were placed in a tubular electric furnace and connected in parallel to one aspirator. The tubes were heated to a bright red heat. The temperature for the different determinations was practically the same, equal currents being always passed through the electric furnace for the same length of time. The tubes were then rapidly but thoroughly flushed until the aspirator was filled down to a certain mark. In order to prevent absorption of the emanation the water in the aspirator bottle was heated by an immersed electric coil. The air containing the emanation was then passed into the ionization chamber through the calcium chloride and phosphorus pentoxide tubes, care being taken to thoroughly flush the tubes with air so that all the emanation would be carried into the ionization chamber. The chamber had been made with the necessary volume to accommodate all the gas driven off from 140 grams of charcoal with a liberal margin for flushing. The electroscope readings were always taken over practically the same region of the scale, the reading being started as nearly as possible thirty minutes after introducing the emanation into the chamber. The deflection of the aluminum leaf for the following thirty minutes was then recorded. By this method the reading was always obtained over approximately the same portion of the decay curve for radium emanation, thereby making the electroscope readings directly comparable.

AMOUNT OF RADIUM EMANATION IN THE ATMOSPHERE AT MANILA.

The theory underlying the calculations of the radium-emanation content from observations by the charcoal-absorption method has been given by several writers on the subject. If M represents the radium equivalent in grams of the emanation in one cubic meter of free air, then

$$M = \frac{M'\lambda T}{V}\frac{d}{d_1},$$

where M^1 is the number of grams of radium in the standard solution, λ the radioactive constant of radium, d the electroscope reading due to the emanation from V cubic meters of air, and d_1 the electroscope reading corresponding to the emanation formed by M^1 grams of radium in the time interval T.

In the deduction of the above equation it is assumed that all the accumulated emanation has been removed from the solution previous to a run, and that all the emanation formed during the time of collecting is removed from the solution and passes through the collecting system. Our preliminary experiments had shown that this assumption is not justified for the conditions of temperature, rate of flow of air stream, etc., under which our tests were made. In which case it is necessary to multiply the above equation by some factor having a value less than unity. If we represent this factor by α the equation then becomes

$$M = \frac{\alpha M^1 \lambda T}{V} \frac{d}{d_1}.$$

A long series of determinations, with conditions maintained as rigidly constant as possible, gave for this factor a value of 0.792. Extreme care was taken throughout all our tests on the emanation content to maintain conditions the same as those existing during the determination of the value of α.

Table I. gives the results of a series of observations for the period from July, 1913, to July, 1914, together with such meteorological data as is necessary to show any existing correlation. The meteorological data is taken from the reports of the Manila Observatory which is situated about 400 meters from the Bureau of Science, consequently the two sets of data practically coincide as to location. All the radioactive determinations given in Table I. were taken under identical experimental conditions. The standard solution was put into the "steady state" by bubbling air at the rate of 0.5 liter per minute through the boiling solution for one hour and then through the cool solution for two hours. At the end of the two hours the air stream was started through the collecting system, the rate of flow being maintained constant at 0.5 liter per minute. All observations extended from 1 P.M. to 9 A.M., a period of twenty hours, making the volume of air tested in every case equal to 0.6 cubic meter. As will be seen later it is extremely important that the observations should be taken over the same part of the day if the results are to have a high comparative value.

TABLE I.

Variation of the Radium Emanation Content with Meteorological Conditions.

Date.	Pressure (Mean), (Mm.)	Nature of Variation of Pressure During the Three Days.	Humidity (Mean), (%).	Wind.			Rain, 24 Hours Beginning Midnight, (Mm.).	Rain for the Three Days, (Mm.).	Radium Emanation per Cu. M. Expressed in its Radium Equivalent, (Grams $\times 10^{12}$).	Weather Remarks.
				Prevailing Direction.	Total Movement, (Km.).	Total Movement During the Three Days, (Km.).				
July 8, 1913	757.94		80.0	WSW	267.0		....			Fair during July 8 and during day of July 9. Very heavy shower during night of 9th.
July 9, 1913	757.60	Decreasing	81.6	WSW	321.0	922.0	11.8	13.3	35.8	
July 10, 1913	757.47		92.9	WSW	334.0		1.5			
July 22, 1913	755.30		91.4	WSW	468.0		28.5			Very heavy rains during the past 10 days.
July 23, 1913	753.64	Variable	89.3	WSW	564.5	1381.0	14.1	45.0	20.5	
July 24, 1913	753.91		89.4	SSW	348.5		2.4			
July 27, 1913	756.90		88.2	WNW	218.0		13.8			Very heavy rains during the past 18 days.
July 28, 1913	753.74	Decreasing	91.0	WSW	476.0	1742.0	5.0	72.7	14.5	
July 29, 1913	750.80		90.1	SW	1048.0		53.9			
Aug. 3, 1913	757.80		86.1	SW	290.0		1.3			Heavy typhoon weather for preceding three weeks, but only partly cloudy with occasional showers beginning with Aug. 2.
Aug. 4, 1913	757.12	Decreasing	93.0	SW	332.0	830.5	23.0	24.7	23.6	
Aug. 5, 1913	756.36		89.7	SW	208.5		0.4			
Aug. 17, 1913	758.60		85.9	SE	109.5		9.7			
Aug. 18, 1913	759.26	Variable	87.7	Variable	94.0	367.5	1.8	54.7	39.5	
Aug. 19, 1913	757.26		86.0	SW	164.0		43.2			

TABLE I.—*Continued.*

Date.	Pressure (Mean), (Mm.).	Nature of Variation of Pressure During the Three Days.	Humidity (Mean), %	Wind.			Rain, 24 Hours Beginning Midnight, (Mm.).	Rain for the Three Days, (Mm.)	Radium Emanation per Cu. M. Expressed in its Radium Equivalent, (Grams $\times 10^{12}$).	Weather Remarks.
				Prevailing Direction.	Total Movement (Km.).	Total Movement During the Three Days, (Km.).				
Aug. 24, 1913	757.51		88.6	Variable	246.5		16.5			Partly fair during Aug. 22 and 23. Heavy showers during night of 24th.
Aug. 25, 1913	758.77	Variable	88.8	SW	335.5	979.0	5.3	29.1	19.6	
Aug. 26, 1913	757.34		89.0	SW quad.	397.0		7.3			
Aug. 31, 1913	757.98		84.3	SW	224.0		19.2			Fair during the day time of Aug. 31, but heavy showers during the night.
Sept. 1, 1913	755.98	Decreasing	83.0	W quad.	149.0	652.0	0.8	25.1	49.7	
Sept. 2, 1913	753.53		85.9	SW	279.0		5.1			
Sept. 7, 1913	758.48		84.0	WSW	271.0		....			Occasional heavy showers during the period from Sept. 4 to Sept. 8.
Sept. 8, 1913	757.74	Decreasing	81.2	WSW	530.0	1472.0	....	105.8	17.3	
Sept. 9, 1913	756.39		88.8	SW, WSW	671.0		105.8			
Sept. 21, 1913	757.76		82.9	W quad.	117.5		0.8			Light afternoon showers for the period from Sept. 18 to Sept. 21 inclusive.
Sept. 22, 1913	758.62	Increasing	82.0	SW quad.	159.0	392.0	....	0.8	53.4	
Sept. 23, 1913	759.83		87.8	SW quad.	115.5		....			
Sept. 28, 1913	759.57		82.4	W quad.	136.0		....			Light showers during the preceding week with no heavy rains.
Sept. 29, 1913	759.16	Decreasing	84.8	W	113.0	390.5	5.1	12.0	52.0	
Sept. 30, 1913	758.46		82.4	W quad.	141.5		6.9			
Oct. 12, 1913	753.95		88.6	N quad.	124.0		4.7			Occasional heavy showers with considerable fine drizzling rain intervening during Oct. 10 and 11.
Oct. 13, 1913	751.87	Variable	91.6	SSW	459.0	1067.0	13.1	66.3	17.1	
Oct. 14, 1913	752.36		93.3	SSW	484.0		48.5			

TABLE I.—*Continued.*

Date.	Pressure (Mean), (Mm.).	Nature of Variation of Pressure During the Three Days.	Humidity (Mean), (%).	Wind.			Rain, 24 Hours Beginning Midnight, (Mm.).	Rain for the Three Days, (Mm.).	Radium Emanation per Cu. M. Expressed in its Radium Equivalent, (Grams $\times 10^{12}$).	Weather Remarks.
				Prevailing Direction.	Total Movement, (Km.).	Total Movement During the Three Days, (Km.).				
Oct. 19, 1913	760.24	Variable	81.5	E quad	70.5		0.3			Only very light showers since the 14th. Fair most of the time with light winds.
Oct. 20, 1913	760.60		86.3	SW, NW	55.0	201.5	0.5	0.8	77.1	
Oct. 21, 1913	760.24		83.7	W, WSW	76.0		....			
Oct. 26, 1913	760.28		76.5	NNW, SSW	94.0		....			Fair for the past week with a few light showers and light winds.
Oct. 27, 1913	760.03	Variable	78.4	W	112.0	264.5	....	....	92.2	
Oct. 28, 1913	760.88		81.8	W quad.	58.5		....			
Nov. 2, 1913	760.35		82.4	NW	95.0		....			Fairly heavy local showers during the past week.
Nov. 3, 1913	760.58	Increasing	88.1	W quad.	81.0	248.5	4.6	7.2	49.9	
Nov. 4, 1913	760.61		93.1	NNE	77.5		2.6			
Nov. 9, 1913	760.64		77.5	NE	167.5		....			Considerable rain during the preceding week, but fair on the 9th.
Nov. 10, 1913	760.60	Decreasing	87.2	N quad.	99.0	414.0	3.4	3.4	75.4	
Nov. 11, 1913	760.59		77.6	NE, WSW	147.5		....			
Dec. 7, 1913	762.08		78.5	NE	106.5		....			Light showers on the 5th and 6th. Fair on the 7th.
Dec. 8, 1913	761.22	Variable	79.2	ENE	119.0	373.0	1.8	2.4	59.1	
Dec. 9, 1913	761.39		82.6	NNE	147.5		0.6			

TABLE I.—*Continued.*

Date.	Pressure (Mean), (Mm.).	Nature of Variation of Pressure During the Three Days.	Humidity (Mean), (%)	Wind.			Rain, 24 Hours Beginning Midnight, (Mm.).	Rain for the Three Days, (Mm.)	Radium Emanation per Cu. M. Expressed in its Radium Equivalent, (Grams $\times 10^{12}$).	Weather Remarks.
				Prevailing Direction.	Total Movement, (Km.)	Total Movement During the Three Days, (Km.).				
Dec. 14, 1913	760.13		77.9	NNE	171.0		....			Afternoon showers during the preceding week. Showers during night of 13th.
Dec. 15, 1913	759.85	Variable	83.3	WNW, WSW	80.0	309.0	....	9.3	137.9	
Dec. 16, 1913	760.50		89.5	Variable	58.0		9.3			
Dec. 22, 1913	760.02		79.8	WNW, SE	114.5		....			Fair during the 21st and 22nd. Cloudy during morning of 23rd.
Dec. 23, 1913	760.27	Increasing	86.3	SE	71.5	270.0	3.0	3.0	70.8	
Dec. 24, 1913	760.33		80.2	W quad.	84.0		....			
Jan. 4, 1914	764.59		74.6	E quad.	155.5		....			Fair during the four days immediately preceding.
Jan. 5, 1914	764.34	Decreasing	78.2	E, SE	188.5	538.0	....	....	89.1	
Jan. 6, 1914	764.19		76.2	SE	194.0		....			
Jan. 18, 1914	762.93		72.0	E quad.	169.0		....			Fair.
Jan. 19, 1914	761.90	Decreasing	79.1	E quad.	161.0	531.0	....	....	145.1	
Jan. 20, 1914	761.57		76.1	ESE	201.0		....			
Feb. 1, 1914	761.02		75.0	SE quad.	195.0		....			Fair.
Feb. 2, 1914	761.12	Increasing	76.3	ESE	128.0	427.0	....	....	122.4	
Feb. 3, 1914	761.66		79.2	SE	104.0		....			

TABLE I.—*Continued.*

Date.	Pressure (Mean), (Mm.).	Nature of Variation of Pressure During the Three Days.	Humidity (Mean), (%).	Wind.			Rain, 24 Hours Beginning Midnight, (Mm.).	Rain for the Three Days, (Mm.).	Radium Emanation per Cu. M. Expressed in its Radium Equivalent, (Grams $\times 10^{12}$).	Weather Remarks.
				Prevailing Direction.	Total Movement, (Km.).	Total Movement During the Three Days, (Km).				
Feb. 15, 1913	763.23		74.2	ESE	195.5		....			Fairly heavy showers during the period from Feb. 13 to Feb. 15.
Feb. 16, 1914	763.09	Decreasing	66.4	E quad	229.5	624.5	....	....	94.1	
Feb. 17, 1914	762.96		70.2	ESE	199.5		....			
Mar. 8, 1914	759.23		69.6	ESE	264.5		....			Exceptionally clear and warm during the last few days.
Mar. 9, 1914	759.31	Increasing	70.1	ESE	228.5	755.0	....	....	108.6	
Mar. 10, 1914	759.71		70.6	SE	262.0		....			
Apr. 21, 1914	761.88		71.6	SE	218.5		....			
Apr. 22, 1914	760.51	Decreasing	69.6	NNE, WSW	189.0	610.5	1.5	1.5	68.9	
Apr. 23, 1914	759.87		73.4	SE	203.0		0.3			
May 6, 1914	758.36		644.	SE	206.5		....			
May 7, 1914	759.37	Variable	66.9	SE, W	226.5	630.0	....	2.0	70.1	
May 8, 914	758.96		71.5	E quad.	197.0		2.0			
May 24, 1914	758.52		74.9	WSW	137.5		....			
May 25, 1914	758.66	Variable	78.3	Variable	155.5	499.0	2.3	2.3	97.4	
May 26, 1914	757.02		78.9	WSW	206.0		....			
June 7, 1914	759.41		79.5	Variable	98.0		....			Very heavy shower during the early evening of the 8th.
June 8, 1914	758.97	Decreasing	78.6	WNW	141.0	380.5	6.4	6.4	63.5	
June 9, 1914	758.92		77.7	W	138.5		....			

TABLE I.—*Continued.*

Date.	Pressure (Mean), (Mm.).	Nature of Variation of Pressure During the Three Days.	Humidity (Mean), (%).	Wind.			Rain, 24 Hours Beginning Midnight, (Mm.).	Rain for the Three Days, (Mm.).	Radium Emanation per Cu. M. Expressed in its Radium Equivalent, (Grams $\times 10^{12}$).	Weather Remarks.
				Prevailing Direction.	Total Movement, (Km.).	Total Movement During the Three Days, (Km.).				
June 21, 1914	758.66		85.0	Variable	83.0		....			Very heavy rain during the
June 22, 1914	758.84	Variable	83.8	Variable	103.5	374.0	....	....	54.5	period from June 18 to June 21.
June 23, 1914	758.82		79.5	SE	187.5		....			
July 8, 1914	755.48		90.5	SW quad.	282.0		21.3			Just clearing off after almost
July 9, 1914	756.80	Increasing	84.7	WSW	327.0	1288.0	0.9	22.2	19.2	a week of typhoon weather with
July 10, 1914	757.05		81.6	WSW	679.0		....			very heavy rain and winds. No rain during test.
								Mean	63.4	

The mean value of the radium equivalent per cubic meter of the atmosphere at Manila, as given by the 29 observations recorded in Table I., is 63.4×10^{-12} grams. If we take the mean of all our observations for Manila we obtain a value of 76.7×10^{-12} grams, which is probably nearer the true value. Any mean determined as above, however, is subject to wide fluctuations, since the variation with weather conditions is so great that a wide range of values, even of a large number of determinations, might be obtained by bunching the observations during a definite season of the year. A better idea of the average value is obtained by taking the mean of the monthly means for an entire year. In Table II. the results by months are given, the numbers in brackets in the last column indicating the number of individual observations entering into the value for each month. The mean of the monthly means has a value of 71.0×10^{-12} grams.

TABLE II.

Annual Variation of the Radium Emanation Content.

Month.	Pressure (Mean), (Mm.).	Humidity (Mean), %	Wind, Total Movement for Month, (Km.).	Rain, (Total), (Mm.).	Radium Emanation per Cu. M. Expressed in its Radium Equivalent, Grams $\times 10^{12}$.
July, 1913.....	756.26	86.2	10,374.6	570.6	23.6 [3][1]
Aug., 1913.....	756.93	87.0	8,843.5	349.1	27.6 [3]
Sept., 1913....	757.67	85.4	8,664.5	365.5	43.1 [4]
Oct., 1913.....	758.51	83.4	4,152.0	119.7	62.1 [3]
Nov., 1913.....	761.04	81.7	3,667.5	31.1	62.7 [2]
Dec., 1913.....	761.32	80.9	4,021.0	37.8	89.3 [3]
Jan., 1914.....	763.24	76.2	4,925.5	3.5	117.1 [2]
Feb., 1914.....	762.26	73.8	5,255.5	7.3	108.3 [2]
Mar., 1914.....	760.77	68.6	6,344.0	6.1	108.6 [1]
Apr., 1914.....	760.17	70.8	5,921.0	53.4	68.9 [1]
May, 1914.....	758.42	72.6	6,137.5	84.0	83.7 [2]
June, 1914.....	757.62	81.7	6,714.0	367.9	59.0 [2]
July, 1914.....					19.2 [1]
Mean of monthly means.....					71.0

The following average results have been obtained for the radium-emanation content by Eve[2] at Montreal, Satterly[3] at Cambridge, and Ashman[4] at Chicago:

Eve..... 60×10^{-12} gram Ra. per cubic meter.
Satterly..... 105×10^{-12} gram Ra. per cubic meter,
Ashman..... 96×10^{-12} gram Ra. per cubic meter.

[1] Number in bracket shows the number of observations entering into the monthly mean.
[2] Phil. Mag., *16*, 622, 1908.
[3] Am. Journ. Sci., 119, 1908.
[4] Phil. Mag., *20*, 1, 1910.

Eve and Satterly used the charcoal-absorption method; Ashman the condensation method. Eve's value is the mean of the monthly means for a period from July to April, Satterly's value is the mean of individual observations taken during the months of March to August, inclusive, while Ashman's is the mean of six determinations, the time of year not specified.

In comparing our results with those of Eve and Satterly it should be remembered that they both assumed that all the emanation was obtained from their standard by bubbling air through the cool solution, and consequently did not use in their calculations any factor corresponding to α. If their results are subject to a correction of approximately the value which we found for α, then our mean is considerably greater than that of Eve for Montreal and slightly less than that of Satterly for Cambridge. No great importance is here attached to the fact that our mean value for Manila seems to be nearer to that for Cambridge than Montreal, except in so far as it may throw light on the question as to what extent the value of the emanation content is dependent on the distance that the air tested has traveled over land or water.

VARIATION OF THE EMANATION CONTENT WITH METEOROLOGICAL CONDITIONS.

From a study of the individual observations of Table I. a distinct correlation is observed between the values of the emanation content and certain of the meteorological factors. For each observation of the emanation content we have given the meteorological data for three days, including the two days during which the observation was taken and the preceding day. That a distinct relation exists between the emanation content, the rainfall and the wind movement is readily seen. In every case a period of heavy rains accompanied by high winds corresponds to a low value of the radium-emanation content, while fair weather with light winds gives correspondingly high values. A good idea of the magnitude of this variation is obtained from the ratio of the maximum to the minimum, which has approximately the value of ten to one. Since in Manila the temperature is fairly constant throughout the year the variations in rainfall and wind movement must necessarily be the chief causes of the wide difference in the values of the atmospheric radioactivity. Which one of the two factors is the more important is somewhat doubtful, since they are generally so correlated that it is difficult to determine exactly what part of the total effect is due to either one.

Not only is there a close relation between the emanation content, the rainfall and the wind movement for the individual observations, but also

for the means for the months, as shown in Table II. Taken in conjunction with Table I. it is seen that the observations scattered throughout the month are fairly typical for the entire period. This is especially true for the months in which three or more observations were taken. It is to be regretted that during certain months the rush of other work prevented our taking as large a number of observations as we should have liked. The month of January gives the highest value for the activity, the minimum value for the rainfall, and a low value for the wind movement. July shows the lowest activity, the maximum rainfall and the highest wind movement. A similar relation exists for almost every month of the year. The month of June from Table II. would appear to be an exception, but by referring to Table I. it is seen that the two observations for the month were taken at periods of very little rainfall and comparatively low wind movement, and consequently are not typical of the entire month as far as rain and wind is concerned.

One other factor, namely the direction of the wind, may possibly have an appreciable effect on the value of the emanation content. The location of Manila is such that for the greater part of the year the prevailing wind is from off the sea. Even those which are apparently land breezes, with the exception of those from the northeast quadrant, have at the most passed but a comparatively short distance over land. During the rainy season the prevailing winds are westerly and the radium-emanation content is low, but just as soon as the rainfall and the wind movement decrease the radioactivity increases, even though the direction of the wind remains the same. Certain observations, however, seem to indicate that the highest values are obtained on days when the air currents are from the land, but the results are so uncertain as to practically prohibit the drawing of any definite conclusions. A more detailed discussion of this point will be given in connection with certain observations in which the factor of rainfall is absent.

DIURNAL VARIATION OF THE RADIUM-EMANATION CONTENT OF THE ATMOSPHERE.

In our earlier work on the subject we had noticed that there seemed to be a large variation between consecutive observations taken over different parts of the day. No definite attempt had been made, however, to determine accurately the diurnal variation.

Several observers have investigated the question of the daily variation of the radioactivity of the atmosphere by the active-deposit method. Simpson[1] in Lapland made four determinations a day (3–5 A.M., 10–12

[1] Phil. Trans. Roy. Soc., A-V, CCV., 61, 1905.

A.M., 3–5 P.M., and 8.30–10.30 P.M.) for a period of almost a year. His results showed a maximum for the early morning hours and a minimum about noon, the ratio of the maximum to the minimum being approximately 3 to 1. Gockel[1] at Freiburg likewise found an increase in the early morning and at times a depression about noon. Blackwood[2] at Manila found a maximum about 4 A.M. and a minimum around 6 P.M., the ratio of the means being approximately seven to one. These three observers obtained the active deposit on a negatively charged wire stretched horizontally in free air.

In order to eliminate the variable factor of wind force, Dike[3] used a modification of the above method. The air to be tested was drawn at a definite rate through a wooden pipe over one end of which was fitted a piece of wire gauze. The gauze was well insulated and charged to a high negative potential. The radioactive substances were collected on the gauze and tested in the usual manner. As a mean of six sets of observations, each extending over approximately twenty-four hours, a decided maximum was found in the early morning which decreased to a strong minimum during the early evening. The maximum had a value approximately eighteen times the minimum.

Hess[4] using a method somewhat similar to Dike's found an early morning maximum which decreased to a minimum about 11.30 A.M. rising again to a secondary maximum at 4 P.M. and then rapidly falling to a decided minimum around 6 P.M. The greatest variation found was only about 20 per cent. from the mean value.

From the work of these observers it seems quite certain that the emanation content is considerably greater in the early morning hours than during the late afternoon, even for widely scattered points on the earth's surface. The active-deposit method fails, however, to throw much light on the cause of this variation. Simpson drew the following conclusions from his results:

1. For the whole year temperature has a marked effect, but very little effect during any one month.

2. The radioactivity increases with increase in the humidity and decreases with decreasing humidity.

3. The radioactivity is greater with a falling barometer than with a rising barometer, although the activity is not necessarily higher with a low barometer than with a high one.

4. The radioactivity is greater for winds from the land than for winds from the sea.

[1] Phys. Zeit., 5, 591, 1904.
[2] Phil. Journ. Sci., IX–A, No. 4, 1914.
[3] Terr. Mag., 125, 1906–07.
[4] Sitz. d. k. Ak. d. Wiss., *119*, 145, 1910.

Hess likewise concluded that a decrease in pressure was accompanied by an increase in the active deposit, and vice versa.

The question of whether all these factors directly affect the emanation content or whether certain of them cause merely a variation in the amount of active deposit collected led us to take some observations by the charcoal-absorption method. Since the object of these tests was to determine the causes rather than the nature of the diurnal variation no attempt was made to take observations of extremely short duration.

The first series of tests were taken with the object of determining the relation existing between the emanation content for day and night periods. The day tests extended in every case from 6.30 A.M. to 6.30 P.M., and the night from 6.30 P.M. to 6.30 A.M. The corresponding day and night observations were always made within the 24 hour period in order that the individual readings might have a direct comparative value. The results are given in Table III. With one exception all the night obser-

TABLE III.

Diurnal Variation of the Radium Emanation Content.

Day determinations. Time.—6.30 A.M. to 6.30 P.M.

Night Determinations. Time.—6.30 P.M. to 6.30 A.M.

	Date.	Pressure Variation.	Humidity (Mean), (%).	Wind. Prevailing Direction.	Wind. Total Movement for the 12 Hours, (Km.).	Rain for 12 Hours, (Mm.).	Radium Emanation per Cu. M. Expressed in its Radium Equivalent, (Grams × 10^{12}).
Day Determinations	Dec. 29, 1913	Variable	67.5	Variable	97.0	.....	68.3
	Jan. 2, 1914	"	65.5	N	125.5	.....	51.8
	Jan. 12, 1914	"	70.6	Variable	118.5	.....	66.1
	Jan. 14, 1914	"	76.4	N	85.0	.....	59.5
	Jan. 16, 1914	"	75.4	Variable	65.0	.....	36.2
	Jan. 27, 1914	"	63.4	Variable	127.5	.....	68.0
	Jan. 29, 1914	"	68.2	Variable	99.0	.....	68.0
	Feb. 24, 1914	"	68.4	W. quad.	118.0	.....	85.5
	Mean				104.4	.....	62.9
Night Determinations	Dec. 29–30, 1913	Variable	80.8	NE	75.0	.....	66.9
	Jan. 2–3, 1914	"	80.9	N	27.5	.....	52.4
	Jan. 12–13, 1914	"	89.1	Calm	29.5	.....	165.0
	Jan. 13–14, 1914	"	85.1	Calm	40.5	.....	111.3
	Jan. 15–16, 1914	"	86.0	Calm	21.0	.....	133.8
	Jan. 26–27, 1914	"	79.0	ESE	65.0	.....	108.3
	Jan. 28–29, 1914	"	78.2	ESE, Calm	48.0	.....	174.9
	Feb. 24–25, 1914	"	84.0	SE	32.0	.....	173.5
	Mean				41.1	.....	123.3

TABLE IV.

Diurnal Variation of the Radium Emanation Content.

Day Determinations. Time.—11 A.M. to 5 P.M.

Night Determinations. Time.—11 P.M. to 5 A.M.

	Date.	Pressure Variation.	Humidity (Mean), (%).	Wind.		Rain for 6 Hours, (Mm.).	Radium Emanation per Cu. M. Expressed in its Radium Equivalent, (Grams $\times 10^{12}$).
				Prevailing Direction.	Total Movement for the 6 Hours, (Km.).		
Day	Feb. 27, 1914.....	Decreasing	57.9	E. quad.	98.0	.....	45.8
	Mar. 25, 1914....	Decreasing	42.9	Variable	117.5	.....	26.1
	Mar. 30, 1914....	Decreasing	53.3	WSW,SE	73.5	.....	34.1
	Mean.....				96.3	.....	35.3
Night	Feb. 26–27, 1914..	Decreasing	90.0	Calm	12.5	.....	132.0
	Mar. 24–25, 1914.	Decreasing	78.0	Calm	19.0	.....	116.7
	Mar. 30–31, 1914.	Variable	81.0	ESE, Calm	8.0	.....	104.5
	Mean.....				13.2	.....	117.7

vations gave higher values than those for the day. The ratio of the mean value for the night to the mean for the day is 1.95.

A few observations were also taken in which the time of collecting was shortened to 6 hours, the results of which are given in Table IV. Since all the observers using the active-deposit method seem to agree that the radioactivity reaches a maximum in the early morning hours and falls to a minimum some time during the afternoon we chose for our 6-hour observations the intervals from 11 A.M. to 5 P.M., and 11 P.M. to 5 A.M., respectively. As in the 12-hour observations corresponding day and night tests were always taken within the 24-hour period. Again it is seen that the night values are all consistently higher than the day values, the ratio of the mean for the night to the mean for the day being 3.31.

From the above results it appears quite evident that there is a decided daily variation of the emanation content of the atmosphere at Manila. Moreover, the variation found by the charcoal-absorption method seems to be in good agreement with the results obtained by the active-deposit method. This would seem to indicate that the active-deposit method is fairly reliable for determinations of the variation at any given locality. That observations taken at widely different altitudes would possess the same comparative value does not necessarily follow, since changes in pressure, nucleation, etc., would in that case introduce undetermined factors. The charcoal-absorption method, moreover, lends itself much more readily to a study of the causes of the existing variation.

The data in Tables III. and IV. show quite conclusively the effect of the total wind movement. The observations were all taken during the dry season and the factor of rainfall is entirely eliminated. Without exception the total wind movement for the day observations was considerably greater than for the corresponding night determinations. For the 12-hour tests the total wind movement for the day is 2.54 times that for the night. This is approximately of the same value as the ratio for the radium-emanation content. The fact that the two relations happen to have nearly the same numerical value has probably no direct significance, but it certainly shows that the question of atmospheric circulation, independent of rainfall, has a very decided influence on the variation of the emanation content. For the 6-hour tests the total wind movement for the day is approximately 7.32 times that for the night, or roughly twice the ratio for the emanation content. From these two series of observations we are able to obtain a fair idea concerning the extent to which the high wind movement is responsible for the decrease of the radioactivity of the air. A study of Table I. in the light of the above results leads to the conclusion that at Manila the fluctuation in the total wind movement is probably responsible for almost one half of the observed variation of the radium-emanation content.

It was also thought probable that the direction of the wind during the different parts of the day might have considerable influence on the above variation. A careful study of the hourly data for the periods of observation fails to show, however, any definite relation. Consequently, we are led to the conclusion that for Manila the important factor, in regard to the wind, is not whether it has passed for a long distance over land or sea, but almost entirely one of rapidity of circulation. That the same relation would be found to exist for a point differently situated with regard to water and land does not necessarily follow.

If the circulation of the air rather than its direction is the important question then evidently the decrease of the emanation content due to the wind may be explained as due almost entirely to the mixing of air of low and high emanation content. Moreover, it seems as if all air in rapid circulation is such a mixture, regardless of whether it has passed for a long distance over water or not. This is easily explained if we assume that the air in rapid circulation over land is a mixture of air of high and low altitude, since without doubt the emanation content of the atmosphere decreases with altitude. Only when the air is very still does the amount of emanation it contains begin to approach that exhaled from the ground for the given locality. We are perfectly cognizant that the above explanation may be applicable only to our particular location.

Manila is so situated that practically all land breezes have passed within a short distance over low mountain ranges, which will tend to give a mixture such as assumed above.

The correlation between the radioactive constituents of the atmosphere and the temperature, humidity, and pressure has been the subject of much investigation, but the results are decidedly discordant. For this reason we thought it advisable to include in our tables the data showing the variation in humidity and barometric pressure. The temperature for Manila is so nearly constant for the year that it has been omitted. A careful study of all our results fails to give any convincing evidence that the emanation content is directly dependent to any appreciable extent on either the humidity or the pressure. In the course of this study we plotted the pressure-time curves for the results given in Tables III. and IV., but were absolutely unable to discover any definite relation for either an increasing or decreasing pressure. Since most of the previous investigations on this phase of the question have been made by the active-deposit method it seems probable that whatever relation was found to exist was due to a variation of the collecting distance rather than to any actual change in the emanation content. That the results obtained by the active-deposit method are influenced by the humidity of the air has been shown by the work of Simpson, Blackwood and others.

SUPPLY OF RADIUM TO THE AIR FROM SOIL GAS.

Exhalation.—It has been fairly conclusively shown that the earth's crust is the source of the radioactive substances in the atmosphere. A large amount of work by numerous observers has also demonstrated that the radioactive substances are fairly uniformly distributed throughout the surface of the earth. Consequently it is to be expected that more or less emanation is being exhaled from the earth's surface at every point. That the rate of exhalation will vary with the physical and chemical conditions of the surface is to be expected. In order to determine the effect of weather conditions on the rate of exhalation we made a few tests on the emanation exhaled from the surface of the ground at Manila. As far as we have been able to learn the only direct tests made on the exhalation are those of Joly and Smyth at Dublin. Joly and Smyth,[1] working together, made determinations at two different locations, obtaining as mean values 839×10^{-12} and $4{,}087 \times 10^{-12}$ curie per square meter respectively. Smyth,[2] independently, obtained as a mean of 98 observations, taken at a point about 12 meters from the Geological Laboratory of Trinity College, a value of $2{,}637 \times 10^{-12}$ curie.

[1] Proc. Roy. Dublin Soc., XIII., 148, 1911.

[2] Phil. Mag., 24, 632, 1912.

In order to obtain tests on the emanation exhaled a collecting system somewhat similar to that used by Joly and Smyth was devised. The collector consisted of a cylindrical vessel, 20 cm. in diameter and about 30 cm. deep, open at one end and closed, with the exception of a 4 cm. neck, at the other. The open neck was fitted with a rubber stopper from which portions had been cut to allow the inflow of air. Through the stopper and along the axis of the cylinder was passed a brass tube which supported at its lower end a brass disk, 19.4 cm. in diameter. This disk hung about 1.5 cm. above the open end of the cylindrical vessel. When a test was to be made the collector was placed on a level portion of the campus about 7 meters from the north wall of the Physics Laboratory, and pressed very lightly into the ground. The brass tube was then connected, through the charcoal tubes, to a motor driven pump. When the pump was started the air was drawn down into the cylinder through the openings in the rubber stopper, passing over the edges of the brass disk and along the surface of the ground and then up through the brass tube to the charcoal tubes. It was assumed that by this means practically all the emanation exhaled from the confined portion of the surface would be swept into the collector and carried to the charcoal (see Fig. 1).

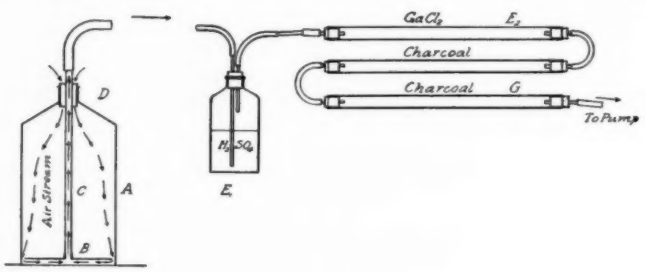

Fig. 1.

A, brass cylinder; *B*, brass disk; *C*, brass tube; *D*, rubber stopper with portions cut out; E_1 and E_2, drying system; *G*, coconut charcoal tubes.

The rate of flow of the air stream was measured by a gauge and was kept fairly constant at about 0.5 liter per minute, the period of collecting being generally one hour. Two charcoal tubes, each containing 70 grams of charcoal, were connected in series. Judging from our preliminary experiments two such tubes should absorb practically all the radium emanation passing through them during a test. The emanation collected in the charcoal was tested in the usual manner.

The results are given in Table V. and are expressed in terms of the radium equivalent of the emanation exhaled in one hour from a surface of one square meter. Although the number of our observations is limited,

TABLE V.

Radium Emanation Exhaled from the Ground.

Date.	Radium Emanation in Air Exhaled per Hour from 1 Square Meter of Ground Surface Expressed in its Radium Equivalent, Grams $\times 10^{12}$.	Weather Remarks.
June 18, 1914..	1642.0	Dry, except for very light showers during the past two weeks.
June 20, 1914..	785.2	Fairly heavy rain during day and night of June 19th. Light rain at intervals during morning of 20th. Light rain during time of collecting.
July 9, 1914...	694.0	Heavy rain and high winds for the last 6 days. A very heavy shower fell for about 15 minutes during the time of collecting.
Aug. 18, 1914..	1122.3	Fair for last three days. A few light showers during the nights. No real heavy rains for almost three weeks.

nevertheless, they show quite plainly the relation of the exhalation for the dry and wet seasons. The observation of June 18 was taken just before the breaking up of the dry season, while that of July 9 followed a week of exceptionally heavy rain. The result of June 18 is 2.37 times that of July 9. This shows approximately to what extent the radioactivity of the air is apt to vary due to rain alone. This is absolutely independent of the question whether rain falling through the air carries down with it any of the emanation. Whether the variations in the wind force effects the rate of exhalation from the ground is still an open question. Since in the above tests the air current remains constant, they throw no light on this phase of the question. The number of our tests is not sufficient to give more than a general idea of the mean value of the radium emanation exhaled at Manila, although our results are in good agreement with those of Joly and Smyth. Without doubt the months of the dry season would show a value as great or greater than that of June 18.

Emanation Content of Soil Gas.—Since the effect of rain is to decrease the rate at which the radioactive emanations are exhaled from the surface of the ground there should be a corresponding increase in the emanation content of the soil gas below the surface. Joly and Smyth found this to be the case for their observations at Dublin, but Satterly[1] failed to discover any correlation between the fluctuations of the emanation content and weather conditions, except for one pipe which always gave low results after rain and high results in fine weather. It was this apparent discrepancy in the results of experimenters working under practically

[1] Proc. Cambridge Phil. Soc., *16*, 514, 1912.

identical conditions which led us to extend our work so as to include a few observations on soil gas.

To obtain the ground air we made three pipes of brass tubing, 9 mm. inside diameter, and 130 cm., 75 cm., and 37 cm. in length. One end of each pipe was fitted with a pointed brass plug. For a length of about 6 cm. from the closed end, holes, sloping slightly downward, were drilled. The pipes were then driven into the ground at a distance of about two meters from a window of the Physics Laboratory to such depths that the mean positions of the portion in which the holes were drilled were approximately 120, 70, and 30 cm., respectively, below the surface of the ground.

The air to be tested was pumped directly into an accurately calibrated gas bottle of one-liter capacity, care being taken in every case to pump out all the air in the pipes previous to taking an observation. Connected in parallel with the gas bottle was a manometer by means of which we could be absolutely certain that the bottle was always filled with gas at atmospheric pressure. After a sufficient time had elapsed to permit the decay of the thorium emanation the air collected was passed into the ionization chamber and tested in the usual manner.

TABLE VI.

Radium Emanation in Ground Air.

Date.	Radium Emanation per Cubic Meter of Ground Air Expressed in its Radium Equivalent, (Grams $\times 10^{12}$).			Weather Remarks.
	Pipe No. 1, Depth 30 Cms.	Pipe No. 2, Depth 70 Cms.	Pipe No. 3, Depth 120 Cms.	
June 15, 1914..			311.5	Dry, except for very light showers, during the period from June 5 to June 15.
June 16, 1914..	32.9	209.1		Generally fair with no rain.
June 17, 1914..	29.4	208.1	269.7	Continued fair weather.
June 19, 1914..	69.5	244.2	284.1	Fairly heavy rain accompanied by high wind during the night of June 18 and continuing during June 19.
June 22, 1914..	44.0	261.6	319.0	Heavy rains up till afternoon of June 20, followed by light showers during June 21 and June 22.
June 24, 1914..	40.7	249.2	314.0	Fair during the last two days. A fairly heavy shower for about 15 minutes when collecting air from Pipe No. 3.
July 14, 1914..	64.0	297.8	321.5	Very heavy rains and high winds during the period from July 4 to July 10 Intervening four days partially fair.
Aug. 19, 1914..	35.1	274.2	314.0	Fair the last four days. A few light showers during the nights. No real heavy rains for almost three weeks.

The results of the observations are given in Table VI. Generally about two hours elapsed between the different observations taken on any given day. It is seen that the values for the 30 cm. pipe fluctuate decidedly with the amount of rainfall, being greater immediately after a period of heavy rains. The 70 cm. pipe shows a slight increase for the rainy weather while the 120 cm. pipe is but little affected. All the results tend to show, however, that the effect of rain is to stop up the soil capillaries, thus retarding the rate of escape of the ground gas, and thereby permitting the emanation in the soil air to approach nearer its equilibrium value.

SUMMARY OF RESULTS.

1. The variation of the amount of radium emanation in the atmosphere at Manila has been determined for a period of about thirteen months. The annual and diurnal variation has been studied in connection with the principal meteorological factors. The effect of weather conditions on the rate at which radium emanation is exhaled from the surface of the ground has been investigated with the object of determining its connection with the emanation content of the atmosphere. The relation between the rate of exhalation and the radioactivity of soil gas at different depths has also been investigated.

2. The variation of the radium-emanation content of the atmosphere has been found to follow quite closely the variations in rainfall and wind movement. The ratio of the maximum to the minimum for the year was found to be approximately as 10 to 1. The mean of the monthly means gives for the radium equivalent of the emanation per cubic meter of air a value of 71.0×10^{-12} grams. The month of January shows the highest monthly mean for the radium-emanation content, the minimum value for the rainfall, and a low value for the total wind movement. The month of July gives the lowest monthly mean for the emanation content, the maximum value for the rainfall, and the highest total wind movement. Every other month of the year shows a very similar relation. No direct connection has been discovered between the emanation content and atmospheric pressure or humidity. The effect of the direction of the wind seems at best very indefinite.

3. A decided diurnal variation has been found to exist, the emanation content being considerably greater during the night than during the day. Observations for the interval from 11 P.M. to 5 A.M. gave a mean value 3.31 times greater than the mean value for the interval from 11 A.M. to 5 P.M. This variation has been found to be closely related to the variation in the total wind movement during the period, a high value of the wind movement corresponding to a low value of the emanation content.

4. The rate at which radium emanation is exhaled from the surface of the ground shows a decided decrease after periods of heavy rain. This decrease has been found in some cases to be almost 60 per cent. of the rate of exhalation for fair weather.

5. The radium-emanation content of soil gas has been determined for depths of 30, 70, and 120 cm., respectively, and the variation with weather conditions studied. The variation in the radioactivity of the gas from the 30 cm. pipe was found to follow closely the variation in the emanation exhaled, a decrease in the exhalation resulting in a corresponding increase in the emanation content of the ground gas. The 70 cm. and 120 cm. pipes showed only slight variations with the weather conditions. The average value of the emanation content for the gas collected from the 120 cm. pipe was found to be 304.5×10^{-12} grams per liter, or over 4,000 times the mean value for atmospheric air. The mean value for the 30 cm. pipe was only about one seventh that for the 120 cm. pipe.

THE CATHODE FALL IN GASES.

By C. A. Skinner.

THIS article is the result of an attempt to develop a theory of the so-called cathode fall in gases, at least to an extent sufficient to suggest intelligent experimental investigation. From the nature of the problem the theory as developed is rather crude. Since, however, it has led to the discovery of certain interesting and quite general relations between the magnitudes involved, which corroborate in part the theoretical deductions, its presentation seemed a logical introduction to these experimental results and necessary to their interpretation.

The theory is based on the generally accepted principles governing ionization by impact and the mobility of the ions, on the one hand, combined on the other with the suggestion made by the writer that the accumulation of the positive ions at the cathode face arises from an elastic rebound of these ions when they impinge on the cathode.[1]

For the sake of clearness we will give first a brief summary of the principal features connected with this cathode fall. Take for example a cylindrical glass tube containing gas at a pressure such that a glow current can be maintained between two disk electrodes placed one in each end of the tube and perpendicular to its axis. For simplicity let the current be confined to the front face of the cathode.

Potential measurements show an extremely low gradient in the negative glow as compared with that at any other portion of the conducting gas. Passing toward the cathode from this region of minimum gradient, one finds that the gradient begins to increase relatively rapidly at a certain point and climbs gradually higher as the cathode is approached.[2]

The negative glow is marked usually by its hazy blue color. The point where the abrupt rise in the gradient begins, is, especially at the higher gas pressures or higher current densities, very distinctly marked as the outer boundary of a bright luminous striation (much whiter than the rest of the negative glow). Towards the cathode this bright striation shades off rapidly to darkness, in the so-called cathode dark space, un-

[1] C. A. Skinner, Phil. Mag., VI., Vol. 4, p. 490, 1902.

[2] This characterization does not include the cases at very low gas pressure in which Graham (Ann. d. Physik, 64, p. 49) observed the gradient to climb in a sinuous manner in passing from the negative glow to the cathode.

accompanied by any correspondingly marked change in the gradient except its gradually increasing magnitude. On the face of the cathode is again a very bright and relatively thin striation.

The potential difference between the cathode and the very low gradient region of the negative glow is known as the "cathode fall." This cathode fall is remarkable in that its magnitude is the same over a wide range of gas pressures, provided the conditions admit of an automatic adjustment of the cathode current density to its so-called normal value.[1] The cathode fall thus obtained is known as the "normal cathode fall."

The potential difference between the cathode and a very fine wire placed only a fraction of a millimeter from it, is, in hydrogen, one half or more of the cathode fall; in nitrogen, it may be a large part of the fall.

If, by means of auxiliary electrodes connected with a constant potential source, one sends a current across the main stream, this cross-current is found to be distinctly larger in the negative glow than at any other part. It decreases especially rapidly as one moves from the negative glow to the cathode. This fact, together with the results of various other experiments which point to the same conclusion, has led to the generally accepted view that the negative glow is very largely the source of the ions which carry the current between that point and the cathode. Thus we have on the cathode side of the negative glow the current carried very largely by the less mobile positive ions moving to the cathode, while to the anode side we have the more mobile electrons. Consequently, on the cathode side we have a greater density of charge, hence a more rapidly increasing potential gradient than on the anode side.

The relatively large potential difference between the cathode and the gas immediately adjacent necessitates the assumption of a greatly decreased mobility of the ions at this point. Many investigators explain this decreased mobility by assuming the existence of a high resistance film on the face of the cathode.

We are interested here however in deducing the results which should arise if the impinging positive ions rebound from the cathode with a definite fraction of their incident energy and only communicate their charge to the cathode (or receive a neutralizing charge from it) when they finally come to rest at its surface. A still more acceptable view would allow for a part of the kinetic energy of the rebounding ion being absorbed by friction in the gas, but the theoretical application of this view appears impossible.

[1] At low gas pressures the minimum cathode fall rises distinctly above this "normal" value, as repeatedly observed during the present investigation. The phenomenon of an automatic adjustment of current density also disappears.

THEORY.

Fall of Potential in the Region between the Negative Glow and Apparent Cathode Film.—Consider a plane cathode perpendicular to the x-axis having a discharge area sufficiently large so that the equipotential surfaces between it and the negative glow are plane and parallel—also the current density uniform.

Throughout that region in which the fall of potential per mean free path, $\bar{\lambda}$, of the electron, is greater than the ionizing potential, we should have, on the average, each electron which leaves the cathode resulting in two electrons at the end of the first mean free path; four at the end of the second; eight, at the third; and so on. With a current density of electrons leaving the cathode equal to $\bar{j}_k$, there should be under a stationary current, at a distance from the cathode $(x_k - x)$, a current density of electrons

$$\bar{j} = \bar{j}_k \cdot 2^{\left(\frac{x_k - x}{\bar{\lambda}}\right)} \tag{1}$$

provided the potential gradient is large enough to cause ionization to take place at each mean free path; and provided, as we may safely assume with such high field strength, the disappearance of electrons by recombination is negligible.[1] Beyond this region of ionization at every mean free path we are under the necessity of assuming another law of ionization. For reasons which will become evident we have assumed the upper limit, namely: that, in the remainder of the path to the point of minimum gradient in the negative glow, ionization occurs on the average whenever an electron traverses a distance over which the fall of potential is equal to the ionizing potential. This is the maximum ionization which could occur under the most favorable conditions of perfectly elastic impact and no recombination.

Between the position x_a where the electrons cease to ionize at each impact and that of the minimum gradient in the negative glow x_0, the number of electrons doubles on moving through the ionizing potential ϵ. Then, since from (1) the negative current density at x_a is

$$\bar{j}_k \cdot 2^{\left(\frac{x_k - x_a}{\bar{\lambda}}\right)}$$

that at any potential V between x_a and x_0 is

$$\bar{j} = \bar{j}_k \cdot 2^{\left(\frac{x_k - x_a}{\bar{\lambda}} + \frac{V - V_a}{\epsilon}\right)} \tag{2}$$

[1] From our measurements of the gradient in hydrogen and Franck and Hertz's value (Verh. d. D. Phys. Ges., *15*, p. 34, 1913) of 11 volts for its ionizing potential, this region of ionization at every mean free path extends nearly to the outer boundary of the cathode dark space.

and the positive current density at the same point necessarily

$$\overset{+}{j} = j - \bar{j}_k \cdot 2^{\left(\frac{x_k - x_a}{\bar{\lambda}} + \frac{V - V_a}{\epsilon}\right)} \tag{3}$$

where j, the total current density, is the same at all parts of the path. We shall use these two equations for obtaining the desired relation between potential and current density, but first it is necessary to obtain $\bar{j}$, $\overset{+}{j}$ and $\bar{j}_k$ in terms of j.

At the position x_0, having potential V_0, (2) gives

$$\bar{j}_0 = \bar{j}_k \cdot 2^{\left(\frac{x_k - x_a}{\bar{\lambda}} + \frac{V_0 - V_a}{\epsilon}\right)}. \tag{4}$$

Poisson's equation,

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = -4\pi\rho,$$

(ρ being the volume density of the charge at the point considered)—applied to the present case, where the equipotential surfaces are plane and parallel, gives

$$\frac{d^2V}{dx^2} = -4\pi\rho. \tag{5}$$

At x_0, dV/dx being a minimum, we have from this equation

$$\rho_0 = 0.$$

That is

$$\overset{+}{\rho}_0 + \bar{\rho}_0 = 0. \tag{6}$$

But

$$\overset{+}{\rho}_0 = -\frac{\overset{+}{j}_0}{\overset{+}{\mu}\left(\frac{dV}{dx}\right)_0} \quad \text{and} \quad \bar{\rho}_0 = \frac{\bar{j}_0}{\bar{\mu}\left(\frac{dV}{dx}\right)_0}, \tag{7}$$

where $\overset{+}{\mu}$ and $\bar{\mu}$ are the mobilities (velocity in unit field) of the positive and negative ions respectively, hence $-\overset{+}{\mu}(dV/dx)_0$ and $\bar{\mu}(dV/dx)_0$ respectively their velocities at x_0.[1] Substituting from (7) in (6) and applying also the necessary condition that

$$\overset{+}{j}_0 + \bar{j}_0 = j$$

we obtain

$$\bar{j}_0 = \frac{\bar{\mu}}{\overset{+}{\mu} + \bar{\mu}} j. \tag{8}$$

Using this last equation to obtain the value of $\bar{j}_k$ from (4) and then substituting that value in (2) and (3)

[1] The signs as used here enable both $\bar{j}_0$ and $\bar{\mu}$ to be treated as positive quantities, $(dV/dx)_0$ being negative.

$$\bar{j} = \frac{\bar{\mu} + \overset{+}{\mu}}{\bar{\mu}} \cdot j \cdot 2^{-((V_0 - V)/\epsilon)},$$
$$\overset{+}{j} = j\left\{1 - \frac{\bar{\mu}}{\bar{\mu} + \overset{+}{\mu}} 2^{-((V_0 - V)/\epsilon)}\right\}. \tag{9}$$

We now proceed to obtain the desired solution. At the position x, having a potential V, equation (5) gives

$$\begin{aligned} \frac{d^2V}{dx^2} &= -4\pi\rho \\ &= -4\pi(\overset{+}{\rho} + \bar{\rho}) \\ &= 4\pi\left(\frac{\overset{+}{j}}{\overset{+}{\mu}\frac{dV}{dx}} - \frac{\bar{j}}{\bar{\mu}\frac{dV}{dx}}\right). \end{aligned} \tag{10}$$

Substituting now in 10 the values of $\bar{j}$ and $\overset{+}{j}$ as given in 9 we obtain the differential equation for the region comprised between the point x_0 of minimum gradient and x_a where the ions cease to ionize at every mean free path:

$$\frac{dV}{dx}\frac{d^2V}{dx^2} = \frac{4\pi j}{\overset{+}{\mu}}\left\{1 - 2^{-((V_0 - V)/\epsilon)}\right\}. \tag{11}$$

Integrating this

$$\frac{dV}{dx} = -\left\{\frac{12\pi j}{\overset{+}{\mu}}[(V_0 - V) - \epsilon(1 - 2^{-((V_0 - V)/\epsilon)})]\right\}^{1/3}, \tag{12}$$

where the condition is introduced that, as shown by the experiment, the gradient dV/dx is practically zero at the position x_0, in the negative glow. A general solution of this equation for V is impossible. Since however the exponential term diminishes rapidly with increasing values of $[(V_0 - V)/\epsilon]$ one may find by trial a value of $(V_0 - V) = q$ such that, if the upper limit of $(V_0 - V)$ be sufficiently high,

$$\begin{aligned} \int_{V_0}^{V}\frac{dV}{dx}dx = \left(\frac{12\pi j}{\overset{+}{\mu}}\right)^{1/3}\Bigg\{&\int_{V_0}^{V_0 - q}(V_0 - V)^{1/3}dx \\ &+ \int_{V_0 - q}^{V}(V_0 - V - \epsilon)^{1/3}dx\Bigg\}. \end{aligned} \tag{13}$$

The following less accurate approximation, however, lends itself more readily to an experimental test of the theory.

From (12) it is obvious that

$$\frac{d}{dx}(V_0 - V) < \left(\frac{12\pi j}{\overset{+}{\mu}}\right)^{1/3}(V_0 - V)^{1/3},$$

also (14)

$$\frac{d}{dx}(V_0 - V) > \left(\frac{12\pi j}{\overset{+}{\mu}}\right)^{1/3}(V_0 - V - \epsilon)^{1/3}.$$

Solving these

$$V_0 - V < \left\{\left(\frac{16\pi j}{\overset{+}{\mu}}\right)^{1/3}(x - x_0)\right\}^{3/2} \tag{15}$$

and

$$V_0 - V > \left\{\left(\frac{16}{9}\cdot\frac{\pi j}{\overset{+}{\mu}}\right)^{1/3}(x - x_0) - \epsilon^{2/3}\right\}^{3/2} + \epsilon \tag{16}$$

for the region between x_0 and x_a.

The right hand side of (15) may be shown to be the value of $(V_0 - V)$ in case x_0 were the sole source of the ions. Then (15) holds for the minimum possible (zero) ionization in the region considered. Hence it holds also for the entire region between negative glow and apparent cathode film.

On the other hand (16) holds, as already stated, for the maximum possible ionization in the region between x_0 and x_a. This same law of ionization, if applied in the region between x_a and the apparent cathode film, would produce a larger ionization than that at every mean free path. It follows then that (16) also holds between the negative glow and the apparent cathode film.

Letting then x_b locate the outer edge of the apparent cathode film and V_b its potential

$$\begin{aligned} V_0 - V_b &< \left\{\left(\frac{16}{9}\frac{\pi j}{\overset{+}{\mu}}\right)^{1/3}(x_b - x_0)\right\}^{3/2} \\ &> \left\{\left(\frac{16}{9}\frac{\pi j}{\overset{+}{\mu}}\right)^{1/3}(x_b - x_0) - \epsilon^{2/3}\right\}^{3/2} + \epsilon. \end{aligned} \tag{17}$$

For convenience of application this expression may be written in another form (in which the mobility $\overset{+}{\mu}$ is assumed proportional to the m.f.p., $\bar{\lambda}$, of the electrons, that is

$$\overset{+}{\mu} = g\bar{\lambda} \tag{17a}$$

g being a constant)

$$\begin{aligned} V_0 - V_b &< \left\{\left[\frac{16\pi}{9g}\right]^{1/3}(j\bar{\lambda}^2)^{1/3}\left(\frac{x_b - x_0}{\bar{\lambda}}\right)\right\}^{3/2} \\ &> \left\{\left[\frac{16\pi}{9g}\right]^{1/3}(j\bar{\lambda}^2)^{1/3}\left(\frac{x_b - x_0}{\bar{\lambda}}\right) - \epsilon^{2/3}\right\}^{3/2} + \epsilon. \end{aligned} \tag{18}$$

The variable quantities are here enclosed in parentheses ( ).

Fall of Potential in the Region of Rebound.—We shall assume that the positive ions in their last mean free path to the cathode are freely acceler-

ated under the action of the electric field. Also that they rebound from the cathode with a fraction κ^2 of their incident energy, and so on with each return, finally transmitting their charge to the cathode simultan-e ously with the disappearance of their kinetic energy of rebound. To just what extent the presence of the gas should alter the results derived from these simpler assumptions, we have not attempted to solve.

The problem is, to find the effectual velocity of the positive ions in the region of rebound and then apply Poisson's equation as in the preceding part.

On its first journey inward each positive ion, having a charge e and mass m, reaches the cathode with a kinetic energy equal to $V_b e$, the potential of the cathode for convenience being assumed zero.

On the first excursion outward the ion rebounds to x_1 at a distance $(x_k - x_1)$ from the cathode such that all of its kinetic energy is converted into potential energy, then it is driven again to the cathode. The potential V_1 of the first turning point is given therefore by the equation

$$V_1 e = \kappa^2 V_b e.$$

Likewise for the second turning point at x_2

$$\begin{aligned} V_2 e &= \kappa^2 V_1 e \\ &= \kappa^4 V_b e, \end{aligned}$$

and so on for the others at x_3, x_4, $\cdots$.

The effectual velocity of the ion at any point x is obtained from the total length of time dt it is between the planes x and $x + dx$.

On its first journey inward (the potential at x being V) the velocity $v_{b,\,k}$ at x is given by the equation

$$\tfrac{1}{2} m v_{b,\,k}^2 = (V_b - V)e,$$

from which

$$v_{b,\,k} = + \sqrt{\frac{2e}{m}} (V_b - V)^{1/2}.$$

On its first excursion out and back it passes the point x twice with a velocity $v_{1,\,k}$ given by the similar equation

$$\begin{aligned} \tfrac{1}{2} m v_{1,\,k}^2 &= (V_1 - V)e \\ &= (\kappa^2 V_b - V)e, \end{aligned}$$

from which

$$v_{1,\,k} = \pm \sqrt{\frac{2e}{m}} (\kappa^2 V_b - V)^{1/2}.$$

Likewise we have

$$v_{2,k} = \pm\sqrt{\frac{2e}{m}}(\kappa^4 V_b - V)^{1/2}$$

$$\vdots \qquad\qquad \vdots$$

and finally

$$v_{p,k} = \pm\sqrt{\frac{2e}{m}}(\kappa^{2p} V_b - V)^{1/2},$$

in which we consider

$$x_p < x < x_{p+1},$$

so that on the pth excursion the ion passes between the planes x and $x + dx$ its last time.

The total length of time dt during which the ion is within the given space dx is then

$$dt = \frac{dx}{v_{b,k}} + \frac{2dx}{v_{1,k}} + \frac{2dx}{v_{2,k}} + \cdots + \frac{2dx}{v_{p,k}}.$$

Its effectual velocity at x is therefore

$$\frac{dx}{dt} = \frac{1}{\frac{1}{v_{b,k}} + \frac{2}{v_{1,k}} + \frac{2}{v_{2,k}} + \cdots + \frac{2}{v_{p,k}}}.$$

The positive current density at x is given by the equation

$$\overset{+}{j} = \overset{+}{\rho}\frac{dx}{dt},$$

from which

$$\overset{+}{\rho} = \frac{\overset{+}{j}}{\frac{dx}{dt}}$$

$$= \overset{+}{j}\left(\frac{1}{v_{b,k}} + \frac{2}{v_{1,k}} + \frac{2}{v_{2,k}} + \cdots + \frac{2}{v_{p,k}}\right).$$

For this vicinity the experimental results indicate that $\bar{j}$ is an extremely small part of j, hence $\bar{\rho}$ is negligible compared with ρ or $\overset{+}{\rho}$. We may then write the last equation

$$\rho = j\left(\frac{1}{v_{b,k}} + \frac{2}{v_{1,k}} + \cdots + \frac{2}{v_{p,k}}\right)$$

$$= \frac{j}{\sqrt{\frac{2e}{m}}}\{(V_b - V)^{-1/2} + 2(\kappa^2 V_b - V)^{-1/2} + \cdots + 2(\kappa^{2p} V_b - V)^{-1/2}\}.$$

Substituting this value of ρ in (5)

$$\frac{d^2V}{dx^2} = -\frac{4\pi j}{\sqrt{\frac{2e}{m}}}\{(V_b - V)^{-1/2} + 2(\kappa^2 V_b - V)^{-1/2} + \cdots + 2(\kappa^{2p} V_b - V)^{-1/2}\}.$$

Integrating

$$\left(\frac{dV}{dx}\right)^2 = \frac{16\pi j}{\sqrt{\frac{2e}{m}}}\{(V_b - V)^{1/2} + 2(\kappa^2 V_b - V)^{1/2} + \cdots + 2(\kappa^{2p} V_b - V)^{1/2}\} + \text{constant} \tag{19}$$

which holds between the turning points p and $p + 1$. This equation cannot be integrated, but remembering that the distance between successive turning points is essentially infinitesimal one may assume the gradient between these points to be linear, its end values given by substituting in this equation.

Letting $V = V_b$ in (19) the integration constant is found equal to

$$\left(\frac{dV}{dx}\right)_b^2,$$

which quantity experiment shows to be negligible when compared with the average value of $(dV/dx)^2$ in the region under consideration.[1] Dropping then $(dV/dx)_b^2$ (19) gives for the first turning point, at x_1, since $V_1 = \kappa^2 V_b$,

$$\left(\frac{dV}{dx}\right)_1^2 = \frac{16\pi j V_b^{1/2}}{\sqrt{\frac{2e}{m}}}(1 - \kappa^2)^{1/2}.$$

For the second turning point, at x_2, $(dV/dx)_1^2$ being now the integration constant,

$$\left(\frac{dV}{dx}\right)_2^2 = \frac{16\pi j V_b^{1/2}}{\sqrt{\frac{2e}{m}}}\{(1 - \kappa^4)^{1/2} + 2\kappa(1 - \kappa^2)^{1/2} + (1 - \kappa^2)^{1/2}\}.$$

For the third, at x_3, $(dV/dx)_2^2$ being the integration constant,

$$\left(\frac{dV}{dx}\right)_3^2 = \frac{16\pi j V_b^{1/2}}{\sqrt{\frac{2e}{m}}}\{(1 - \kappa^6)^{1/2} + 2\kappa(1 - \kappa^4)^{1/2} + 2\kappa(1 - \kappa^2)^{1/2} + (1 - \kappa^4)^{1/2} + 2\kappa(1 - \kappa^2)^{1/2} + (1 - \kappa^2)^{1/2}\},$$

and so on.

[1] For example, in case of the normal cathode fall with an aluminium cathode in hydrogen, the square of the gradient per mean free path of an electron at x_b is about 170, while the square of the average value between x_b and the cathode is about 150,000.

The mean gradients between consecutive turning points, being the average of these end values, may then be written, respectively,

$$\left(\frac{dV}{dx}\right)_{b,\,1} = -\sqrt{\frac{16\pi}{\sqrt{\frac{2e}{m}}}} \cdot j^{1/2} V_b^{1/4} c_{b,\,1},$$

$$\left(\frac{dV}{dx}\right)_{1,\,2} = -\sqrt{\frac{16\pi}{\sqrt{\frac{2e}{m}}}} \cdot j^{1/2} \cdot V_b^{1/4} \cdot c_{1,\,2},$$

and so forth—$c_{b,\,1}$, $c_{1,\,2}$, $\cdots$ being constants whose values depend simply on κ.

Obviously then

$$\int_b^k \frac{dV}{dx} dx = -\sqrt{\frac{16\pi}{\sqrt{\frac{2e}{m}}}} \cdot j^{1/2} V_b^{1/4} \{c_{b,\,1}(x_1 - x_b) + c_{1,\,2}(x_2 - x_1) + \cdots + c_{k-1,\,k}(x_k - x_{k-1})\}. \tag{20}$$

That is

$$V_b = \sqrt{\frac{16\pi}{\sqrt{\frac{2e}{m}}}} \cdot j^{1/2} V_b^{1/4} C(x_k - x_b), \tag{21}$$

in which C is a factor which depends not only on κ but increases with the total number of rebounds, hence should increase with V_b.

We have already assumed that $(x_k - x_b)$ is equal to the mean free path of the positive ions, hence proportional to the mean free path $\bar{\lambda}$ of the electrons. Then

$$(x_k - x_b) = a\bar{\lambda}.$$

Substituting in (21) and writing

$$P = aC,$$

we have for the fall of potential in the apparent cathode film

$$V_b = \left[\frac{16\pi}{\sqrt{\frac{2e}{m}}}\right]^{2/3} \cdot P^{4/3}(j\bar{\lambda}^2)^{2/3}, \tag{22}$$

in which P should be constant with constant values of V_b and κ, but increase with V_b. The first condition allows an interesting test of the theory to be made, namely, for the case under which the cathode fall is found to be constant over a wide range of gas pressures.

The Total Cathode Fall.—Adding (18) and (22) we have the cathode fall

$$V_0 < \left\{\left[\frac{16\pi}{9g}\right]^{1/3}(j\bar{\lambda}^2)^{1/3}\left(\frac{x_b - x_0}{\bar{\lambda}}\right)\right\}^{3/2} + \left[\frac{16\pi}{\sqrt{\frac{2e}{m}}}\right]^{2/3} \cdot P^{4/3}(j\bar{\lambda}^2)^{2/3}$$

$$> \left\{\left[\frac{16\pi}{9g}\right]^{1/3}(j\bar{\lambda}^2)^{1/3}\left(\frac{x_b - x_0}{\bar{\lambda}}\right) - \epsilon^{2/3}\right\}^{3/2} + \epsilon \qquad (23)$$

$$+ \left[\frac{16\pi}{\sqrt{\frac{2e}{m}}}\right]^{2/3} \cdot P^{4/3}(j\bar{\lambda}^2)^{2/3}.$$

These limits for V_0 suggest the simplest condition under which the cathode fall should remain constant with varying gas pressure. For this case the variable quantities in both expressions are

$$(j\bar{\lambda}^2) \quad \text{and} \quad \left(\frac{x_b - x_0}{\bar{\lambda}}\right),$$

the second being, within the errors of measurement, *the number of mean free paths of the electrons between the cathode and the outer edge of the bright striation of the negative glow, where the gradient is a minimum.*

Providing the above limits of V_0 are not too greatly different,[1] V_0 should be constant if $(j\bar{\lambda}^2)$ and $[(x_b - x_0)/\bar{\lambda}]$ are constant.

The results of an experimental study of these magnitudes follow.

EXPERIMENTAL.

Apparatus.—A discharge tube (diam. 3 cm.) was provided with a circular disk cathode (area 3 sq. cm.) accurately adjustable in a direction perpendicular to its face by a screw actuated through a ground joint. A glass hood confined the current to the front face of the cathode. Two fine parallel probe wires (aluminium) lying in the equipotential surfaces, and sheathed to near their ends with very fine glass tubing, served to indicate (in connection with an electrometer) the point of minimum gradient in the negative glow.

It was soon found that, except at the lowest pressures and current densities, the point of minimum gradient, or more exactly the point at which the gradient suddenly began to increase from a zero value, was very closely marked by the luminosity as described on page 000. This being a much more convenient method of setting, it was largely used in the measurements.[2]

[1] Calculations made from observations on an aluminium cathode in hydrogen show these limits of V_0 to differ at most by 12 per cent. (see Table VI.).

[2] Both methods of setting are subject to appreciable error at those low gas pressures where the minimum cathode fall begins to run above its so-called normal value.

Special care was taken to prevent, by sufficient insulation, any leakage current from the probe wires, as the corresponding fall of potential, if the wire for the leakage current is a cathode, has a very marked effect on the indicated value.[1]

The electric current was furnished by a battery of small storage cells, was measured by either a milli-ammeter or a more sensitive galvanometer, and regulated by a solution of cadmium iodide in amyl alcohol.

The cathode fall was measured by a Kelvin electrometer, being placed between the needle and one pair of quadrants, while a definite potential (10–20 volts) was placed between the quadrant pairs.

Normal Cathode Fall, Current Density and Distance to Negative Glow.—The gas chosen was hydrogen because of the relatively large distance it gives between the cathode and negative glow. The metals chosen were aluminium and steel which have respectively (among the common metals) the smallest and largest values of the cathode fall.

The hydrogen was generated from aluminium in a solution of potassium hydrate, then carefully dried in a chamber containing phosphorous pentoxide, and finally stored in a convenient bulb (freed of occluded gases), from which the supply was drawn as needed.

The cathode was always polished with infusorial earth and rubbed clean with new cloth just before mounting in the discharge tube. In addition its discharge surface was always cleaned by a heavy current (passed through the old gas just before evacuating and introducing the fresh). All measurements were made as soon as possible after this last cleaning was done. For each day's observations the cathode was always polished anew.

Tables I. and II. give a representative series of measurements for the two metals respectively. Each recorded pressure (first column) represents an entirely fresh gas filling.

With gas pressures ranging from one to five millimeters the normal cathode fall (second column) fluctuates irregularly about a constant value.

The normal current density (third column) was obtained either by increasing the current to the point where the cathode fall began to increase, or until the negative glow appeared of uniform intensity over the face of the cathode and did not curl away from it at any part—the latter method of setting being usually the more sensitive.

The distance from cathode to the position of minimum gradient was measured by the number of turns of the screw required to shift the cathode from contact with the fixed probe wire to where the wire was at the point of minimum gradient, located as already described.

[1] In order to eliminate errors arising from leakage all lines to the electrometer including the battery used to charge the quadrants, and the electrometer itself had to be placed on sealing wax insulators—no "ground" to the electrometer being used.

TABLE I.

Aluminium Cathode in Hydrogen. Area of Cathode: 3.0 sq. cm.

Gas Pressure (Mm.).	Normal Fall (Volts).	Normal Current Density (j_n)	Distance to Neg. Glow ($x_k - x_0$).	Mean Free Path ($\bar{\lambda}$)	($j_n\bar{\lambda}^2$)	$\left(\frac{x_k - x_0}{\bar{\lambda}}\right)$
1.12	205	.10 m.a.	9.6 mm.	.64 mm.	.041	15
1.65	196	.19	6.8	.44	.037	15.5
2.27	194	.42	4.6	.315	.042	14.6
2.28	196	.43	4.5	.315	.043	14.3
2.41	206	.43	4.7	.30	.039	15.7
3.13	195	.67	3.6	.235	.037	15.3
3.58	194	.90	—	.20	.036	—
4.18	194	1.40	2.7	.17	.040	15.8
5.36	197	—	2.15	.135	—	15.9
Mean	197			Mean	.039	15.3
Max. dev.	4.5%			Max. dev.	10%	6.5%
Mean dev.	3%			Mean dev.	5%	3%

The mean free path of the electrons (column 5) which is inversely proportional to the gas pressure was obtained by multiplying the m.f.p. of the molecule by $4\sqrt{2}$.
ments of this magnitude.

Columns six and seven give respectively the calculated values of the quantities $(j\bar{\lambda}^2)$ and $[(x_k - x_0)/\bar{\lambda}]$ which practically $= [(x_b - x_0)/\bar{\lambda}]$.

The results show that, with constant cathode fall, gas pressures ranging from one to four times, and the normal current densities ranging from one

TABLE II.

Steel Cathode in Hydrogen. Area of Cathode: 3.0 sq. cm.

Gas Pressure (Mm.).	Normal Fall (Volts).	Normal Current Density (j_n).	Distance to Neg. Glow ($x_k - x_0$)	Mean Free Path ($\bar{\lambda}$)	($j_n\bar{\lambda}^2$)	$\left(\frac{x_k - x_0}{\bar{\lambda}}\right)$
1.30	310	.087 m.a.	11.7	.55	.0265	20.1
1.48	306	.10	10.2	.49	.024	20.8
1.73	305	.167	8.4	.42	.0295	20.0
1.95	313	.207	7.6	.37	.0285	20.5
2.11	306	.25	7.1	.34	.029	20.9
2.43	305	.300	5.7	.30	.027	19.0
2.72	305	.367	5.5	.265	.025	20.8
2.93	302	.467	5.1	.245	.028	20.8
3.31	298	.567	4.6	.215	.026	21.4
3.72	298	.767	4.1	.195	.029	21.0
4.00	302	.85	3.7	.18	.0275	20.6
4.77	289	1.20	2.9	.15	.027	19.3
5.00	304	1.33	3.0	.145	.028	20.8
Mean	303.5			Mean	.0273	20.5
Max. dev.	5%			Max. dev.	12%	7%
Mean dev.	2%			Mean dev.	5%	2½%

to sixteen, both of these magnitudes fluctuate irregularly about constant values—the deviation falling on the average well within the errors of observation.

Thus we find, as suggested by the foregoing theoretical deductions, that the *normal current density is inversely proportional to* $\bar{\lambda}^2$, *that is, proportional to the square of the gas pressure*[1]—nearly one and a half times as large with aluminium as it is with steel. Further, *the normal cathode fall extends from the cathode, at all gas pressures, the same number of mean free paths* of the electron. With a steel cathode this number is one and a third times that with an aluminium cathode, its fall being one and a half times as large.

These experimental results being favorable to the theory a more thorough investigation of its appplicability was desirable.

The Potential Curve in the Gas.—Preliminary measurements of the potential difference between the cathode and various distances $(x_k - x)$ in the gas (out to the negative glow), at various pressures and current densities, indicated that for the same values of $(j\bar{\lambda}^2)$ and $[(x_k - x)/\bar{\lambda}]$ the potential at x was always the same. That is, the potentials in the gas plotted against the number of mean free paths from the cathode, fell, for constant values of $(j\bar{\lambda}^2)$, on the same curve.

A systematic investigation of this was carried out with an aluminium cathode in hydrogen. The results are incorporated in Tables III., IV. and V. Each series of observations were made with fresh gas and a current-cleaned cathode. A series consisted in measuring the difference of potential between the cathode and a single probe wire (al., diam. .24 mm.) when the latter was placed at two, three, four, etc., mean free paths (measured to the center of the probe wire) from the cathode—the current density during that time being maintained at "normal," "twice-normal," or "four-normal" value. A disturbing source of error arises from the change in the gas, with duration of current, causing an increase in the potential difference. We are inclined to attribute this change to the presence of metal vapor from the cathode, as its effect is appreciably smaller with aluminium than with steel, and it is well known that the former gives off distinctly less material by the cathode spray than the other metals.

To avoid this error the observations were hurried through as rapidly as possible by three observers—one regulating the current, the second observing the electrometer deflection (to one side only), and the third setting the probe wire.

[1] H. A. Wilson (Phil. Mag., Vol. VI, 4, p. 608) from a study of the current density with wire cathodes in air, concluded that at a determinate distance from the cathode surface the current density is proportional to the gas pressure.

TOTAL RADIATION FROM METALS.

BY VERNON A. SUYDAM.

FROM a consideration of the experimental results thus far obtained the statement is justified that with many substances total radiation may be expressed with fair accuracy by a simple relation of the form

$$E = c'T^n. \tag{1}$$

In the case of some metals and alloys, however, this equation does not express the radiation accurately owing to irregularities in emission. In those cases where an equation of this form does hold n is not equal to 4, as Stefan supposed, except perhaps in exceptional cases. The character of the surface of the body and its internal structure are factors which affect radiation, and, as these vary with the temperature, it is to be expected that the emission cannot always be expressed so simply. From an application of Kirchhoff's law, which may be written

$$\frac{f(T)}{cT^4} = a(T), \tag{2}$$

coupled with the fact that $a(T)$ is a function of the temperature in most cases, it is easily seen that $f(T)$ is not equal to $c'T^4$. $f(T)$ may be of the form $c'T^n$, but n cannot be equal to 4 unless $a(T)$ is constant. The work of E. Hagen and H. Rubens shows $a(T)$ in the case of metals to be a function of the temperature for infra-red radiation, as it is made to depend upon the electrical resistance.[1] The coefficient of electrical resistance for metals is positive when the metal is in the solid state, and hence $a(T)$ increases with T, approaching unity as a limit. Hence metals approach black-body radiation with increase in temperature. The present investigation shows that glass, brass and lamp-black are better absorbers at high than at low temperatures. In the case of all the metals tested n was found to be greater than 4. This result agrees with the observations of F. Paschen and also E. Aschkinass.[2]

METHOD OF OBSERVATION.

The metal to be tested was in the form of a wire which was stretched coaxially in a cylindrical enclosure, a light spring being used to keep it

[1] Phil. Mag., 7, p. 157, 1904. Preuss, Akad. Wiss. Berlin Sitz. Ber., 23, p. 467, 1910.
[2] Drude Ann., 17, p. 960, 1905.

taut when heated. Fine platinum potential-wires were fused to the test-wire at points well removed from its points of entrance to the enclosure. In the case of silver the potential wires were silver soldered. The watt-input was calculated from the fall in potential between the points p_1p_2, Fig. 1 (measured with a low-resistance Leeds and Northrup potentiometer), and the current flowing. The current was calculated

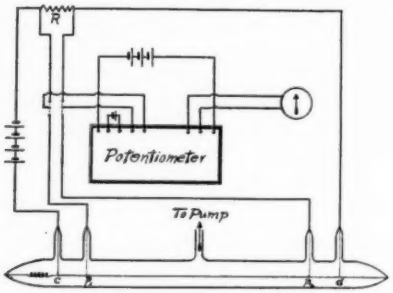

Fig. 1.

from the fall in potential over a known standard resistance, R, placed in series with the test-wire. The arrangement of the apparatus is indicated in Fig. 1.

The temperature of the wire, for any given steady current, was determined from its resistance, which was calculated from the known current and fall in potential between the points p_1p_2. In order that the resistance

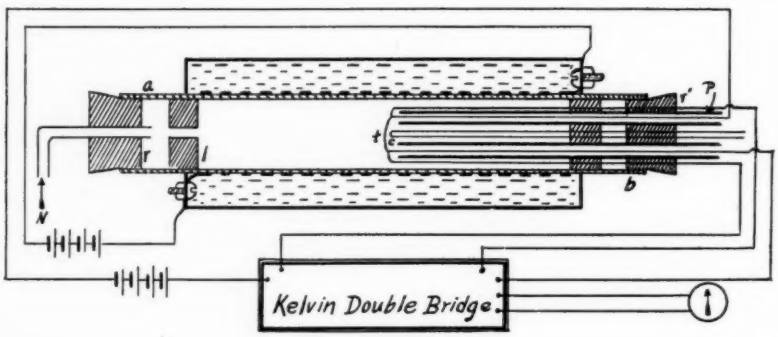

Fig. 2.

of the wire might serve as a measure of its temperature it was necessary to obtain data for a temperature-resistance curve, from which the temperature corresponding to any given resistance, could be determined. The arrangement of the apparatus for obtaining such data is indicated in Fig. 2.

A long quartz tube, *ab*, wound with nichrome ribbon and packed in sand served as a heating furnace. The nichrome was protected from

oxidation by being covered with a layer of alundum cement. With this arrangement the furnace could be heated to 1500° K. repeatedly. The ends of the quartz tube were sealed with rubber corks, rr'. To protect the rubber corks from the heat of the furnace lavite plugs, l, were inserted at a short distance from the corks. This gave sufficient protection for the temperature attained. Through the cork r' and the corresponding lavite plug were passed two porcelain tubes, P, and a Pt *vs.* Pt + 10 per cent. Rh thermocouple. This thermocouple was calibrated by comparing it with a standard thermocouple which had been calibrated at the Bureau of Standards. The wire, t, whose resistance was to be measured was placed in the center of the furnace with the hot junction, c, of the thermocouple close against it. The current terminals entered the furnace through the porcelain tubes, the potential terminals just outside a Leeds and Northrup Kelvin-double-bridge being used to measure the resistance. The wire was protected from oxidation by passing nitrogen through the furnace. The nitrogen was obtained after the method of G. A. Hulett.[1] This method consists in heating copper and copper oxide held in a hard-glass tube, and passing air and hydrogen through the tube. The hydrogen combines with the oxygen of the air and the emergent gas is pure nitrogen.

The metals tested were silver, platinum, nickel and iron, and the alloy nichrome. When platinum was tested the enclosure was made of brass blackened within with lamp-black. For all the other samples a glass enclosure was used. In all cases the surface of the test-wire was carefully cleaned and polished. Before taking a set of observations the wire was heated, by sending a strong current through it, in order to drive off accluded gases.

When the test-wire was platinum the temperature of the enclosure was first held at 90° K., by immersing it in liquid air, then at 273° K., by packing it in melting ice, and then at 373° K., by immersing it in boiling water. In other cases only the temperatures 273° K. and 373° K. were used as fixed temperatures. It was found that the temperature of the enclosure had an influence upon the watt-input. In every case the energy-temperature curve obtained by holding the enclosure at a low temperature was steeper than when it was held at a higher temperature, indicating that the reflecting power of the enclosure increased with decrease in temperature, or $a(T)$ increased with increase in temperature. A. Schleiermacher[2] in his report on radiation from platinum (his method of experimentation was the same as the one here used) observes that the

[1] Jour. of Am. Chem. Soc., 27, p. 1415.
[2] Wied. Ann., 26, p. 287, 1885.

curves meet at a temperature somewhat above that of the highest temperature of the enclosure and states that they coincide above this temperature. The author found in his work that the curves cross but do not coincide at any temperature above the crossing point although they do not separate widely. Data are given here for only two cases, platinum and silver, where the temperature of the enclosure was other than 273° K., as it was found that the influence of the enclosure was negligible for temperatures of the test-wire large in comparison with that of the enclosure.

DISCUSSION.

The watt-input, which is measured, is the difference between the true emission from the metal when at a given temperature and the energy which it receives from the enclosure. This latter is made up of two parts: energy emitted by the enclosure and reflected from it. This action may be represented by the following functional equation:

$$f(T) = \omega(T) + a(T)\{r(T, T_e) + e(T_e)\}, \tag{3}$$

where $f(T)$ is the total radiation from the metal, $\omega(T)$ the watt-input, $a(T)$ the absorption coefficient, $r(T, T_e)$ the reflected energy, which depends upon the temperature, T, of the test-wire and the temperature T_e, of the enclosure, $e(T_e)$ the total radiation from the enclosure, constant for any given set of observations.

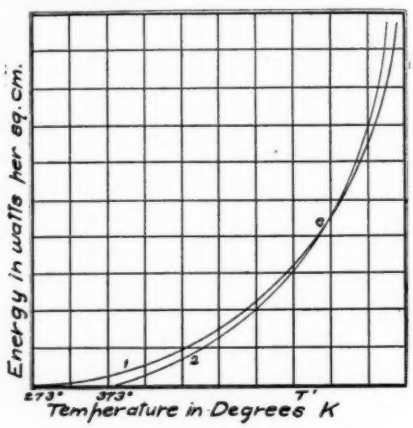

Fig. 3.

Referring to the accompanying diagram, Fig. 3, which is a representation of the curves obtained in this investigation, it is seen that at the point e, where the curves cross at temperature T'

$$\omega_1(T') = \omega_2(T'), \tag{4}$$

where the subscripts refer to the curves obtained when the enclosure was

at 273° K. and 373° K. respectively. From equations (3) and (4) we obtain

$$r(T', T_{e_1}) - r(T', T_{e_2}) = e(T_{e_2}) - e(T_{e_1}). \quad (5)$$

The right-hand member of this equation is constant and depends only upon the substance and temperature of the enclosure. Since

$$e(T_{e_2}) > e(T_{e_1})$$

it follows that

$$r(T', T_{e_1}) > r(T', T_{e_2}).$$

That is, the quantity of energy reflected per second per unit area from the enclosure is greater when it is held at 273° K. than when held at 373° K. Thus in the case of all the enclosures used $a(T)$ increased with increase in temperature over the temperature range used in this investigation.

If there were no energy reflected from the enclosure the slope of the energy-temperature curve would be greater at every point with the

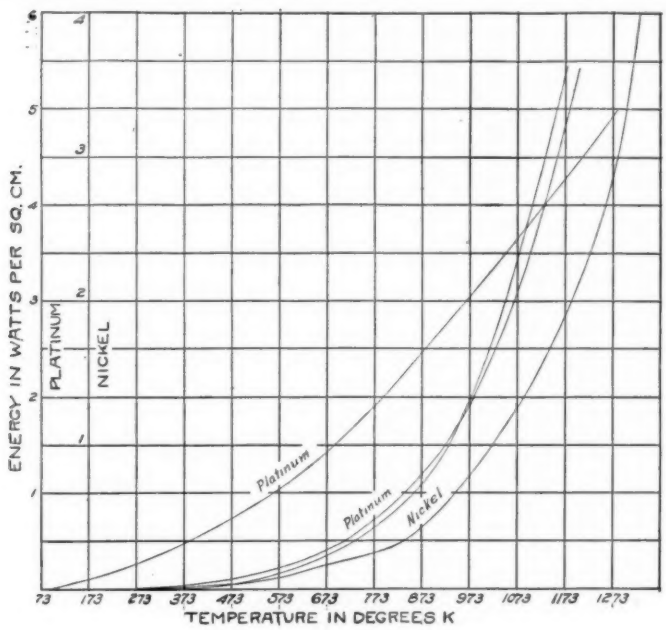

Fig. 4.

Energy-Temperature Curves.

temperature of the enclosure at 273° K. than at 373° K., because less energy would be sent to the test-wire from the enclosure. Since the reverse is the case, it follows that the enclosure is a better reflector at a low than at a high temperature, and hence that the reflected energy

plays a larger part in maintaining the temperature of the test-wire than does the emitted energy. This accounts for the crossing of the curves. That the curves do not separate more widely above the point of crossing seems to indicate a falling off of the percentage of reflected energy for higher temperatures of the test-wire. This may be accounted for from the fact that the energy emitted from the test-wire contains an ever increasing amount in short waves, and as the short waves are more readily absorbed and transmitted there will be a decrease in the percentage of reflected energy.

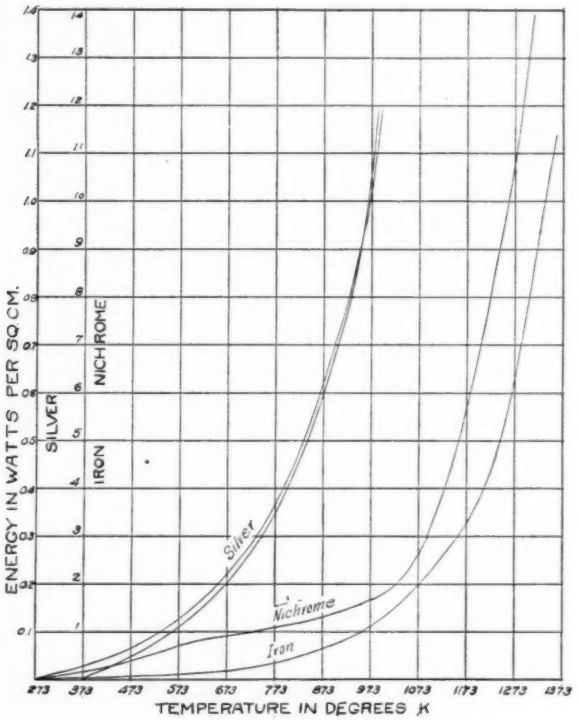

Fig. 5.

Energy-Temperature Curves.

Fig. 4 shows the energy-temperature curves obtained by holding the temperature of the enclosure at 90° K., 273° K. and 373° K. The test-wire was platinum and the enclosure was brass blackened within with lamp-black. It will be noticed that the slope of the curve in each case is greater when the temperature of the enclosure was maintained at a high temperature than when at a lower temperature. It requires less energy to maintain the temperature of the test-wire at, say, 1,133° K., with the enclosure at 90° K. than with the enclosure at 373° K. In the

former case the energy required was 4.02 watts per sq. cm. and in the latter 4.51 watts per sq. cm.

From equations (2) and (3) we obtain

$$\omega(T) = f(T)\left\{1 - \frac{r(T, T_e) + e(T_e)}{cT^4}\right\}. \tag{7}$$

This equation shows that when $T = Te$

$$cT^4 = r(T, T_e) + e(T_e),$$

since $\omega(T) = 0$ and $f(T) \neq 0$. This is a statement for black-body radiation in an enclosure in thermodynamic equilibrium. In the development of this equation no account has been taken of extraneous radiation that might enter the enclosure from without, and it has been

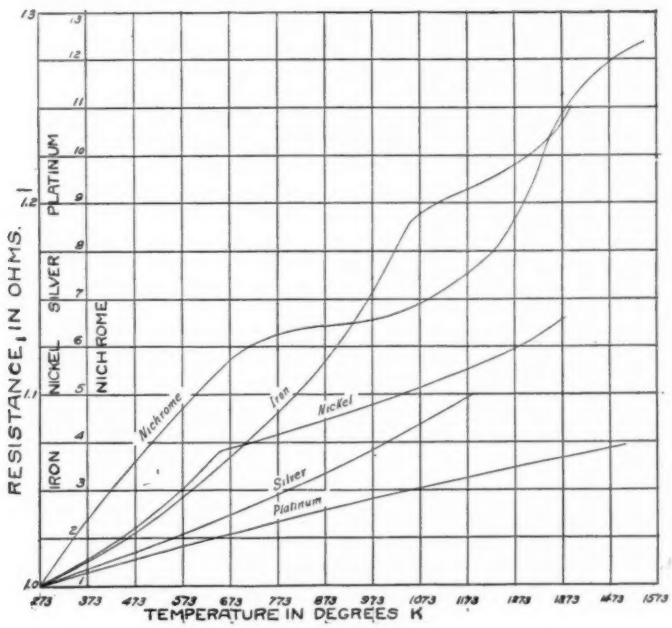

Fig. 6.

Resistance-Temperature Curves; reduced to one ohm at 273° K.

assumed that the enclosure reflects and emits energy as though equal in area to that of the test-wire.

In order to calculate approximately the relative magnitudes of $\omega(T)$ and $f(T)$ in equation (7) it is necessary to evaluate the terms in brackets. L. Graetz[1] found that glass conforms very nearly to black-body radiation for temperatures lying between 273° K. and 456° K., and gives as the average value for c

[1] Wied. Ann., 11, p. 913, 1880.

$$4.52 \times 10^{-12} \frac{\text{watt}}{\text{cm.}^2 \text{ sec. temp.}^4}.$$

For a black-body c is

$$5.65 \times 10^{-12} \frac{\text{watt}}{\text{cm.}^2 \text{ sec. temp.}^4}.$$

Using these values we obtain, by substituting in the last term of the right-hand member of equation (7),

$$\frac{e(T_e)}{cT^4} = \frac{4.52 \times 10^{-12} \times 273^4}{5.65 \times 10^{-12} \times 950^4} = 0.00545,$$

the temperature 950° K. being taken from Table II. Applying Kirchhoff's law to this case we find

$$\frac{e(T)}{cT^4} = \frac{4.52 \times 10^{-12} T^4}{5.65 \times 10^{-12} T^4} = 0.80 = a(T).$$

This gives 20 per cent. for the reflecting power of glass in the temperature interval here used. As a first approximation we may take $\omega(T)$ as equal to $f(T)$. Referring to Table II., the watt-input for silver at

TABLE I.

Platinum.

$T_e = 90°$ K.		$T_e = 273°$ K.		$T_e = 373°$ K.	
Temp. in Deg. K.	Emission in Watts per Sq. Cm.	Temp. in Deg. K.	Emission in Watts per Sq. Cm.	Temp. in Deg. K.	Emission in Watts per Sq. Cm.
115	0.0291	541	0.1540	534	0.1271
126	0.0472	596	0.2323	572	0.1721
144	0.0734	647	0.3293	642	0.2586
201	0.1682	698	0.4562	699	0.3954
283	0.2650	772	0.7104	725	0.4886
351	0.4500	834	0.9776	802	0.7233
487	0.7557	879	1.2477	870	1.0522
567	1.0000	914	1.4726	917	1.3845
632	1.2527	981	2.0338	944	1.6741
723	1.6490	991	2.1597	979	2.0582
786	1.9905	1,018	2.4129	1,007	2.3975
863	2.4250	1,046	2.7412	1,044	2.8930
914	2.7746	1,091	3.3427	1,048	2.9482
1,134	4.0079	1,118	3.7820	1,100	3.7880
1,166	4.2512	1,148	4.2532	1,135	4.4696

$T = 950°$ $K.$ is found to be 0.4903 watt per sq. cm. 20 per cent. of this is 0.098, which is the amount returned to the wire, $r(T, Te)$. Substituting this value in the term next to the last in the right-hand member of equation (7) we find

$$\frac{r(T, T_e)}{cT^4} = \frac{0.098}{5.65 \times 10^{-12} \times 950^4} = 0.0218.$$

The substitution of these values in equation (7) gives a first approximate value for $f(T)$. Thus

$$\omega(T) = 0.973\, f(T).$$

Other errors for which no corrections have been made would change this relation slightly. There is a small loss of energy, due to conduction along the potential wires and in the residual gas and volitalization of the metal, which tends to make $\omega(T)$ too large.

To a close approximation, therefore, $\omega(T)$ may be taken as equal to $f(T)$ for values of T large in comparison with Te. Thus for high temperatures the watt-input, $\omega(T)$, is a function of the same form and of the same order of magnitude as $f(T)$. A. Schleiermacher concludes from the fact that the curves nearly coincide for large values of T that the reflection and radiation from the walls of the enclosure are relatively unimportant.

RESULTS.

Silver.—The diameter of the wire used was 0.0128 cm., and the length between potential points was 44 cm. Silver was found to conform to

TABLE II.

Silver. $T_e = 273°$ K. Average Pressure = 0.0002 Mm.			Silver. $T_e = 273°$ K. Average Pressure = 0.0002 Mm.		
Temp. in Deg. K.	Emission in Watts per Sq. Cm.	c'.	Temp. in Deg. K.	Emission in Watts per Sq. Cm.	c'.
610	0.0859	3.26×10^{-13}	625	0.0875	3.01×10^{-13}
686	0.1313	3.09×10^{-13}	697	0.1306	2.88×10^{-13}
738	0.1735	3.02×10^{-13}	749	0.1741	2.99×10^{-13}
805	0.2435	2.97×10^{-13}	805	0.2595	3.16×10^{-13}
885	0.3693	3.05×10^{-13}	869	0.3279	2.92×10^{-13}
950	0.4903	3.03×10^{-13}	901	0.3927	3.01×10^{-13}
			945	0.4739	3.00×10^{-13}
			981	0.5691	3.08×10^{-13}

the fourth-power law more nearly than any of the other metals tested. Its emission may be represented very accurately by the equation

$$E = c'T^{4.1},$$

where c has the mean value of 2.91×10^{-13} when the temperature of the enclosure was 373° K., and 3.07×10^{-13} when it was 273° K. The enclosure was glass. Only one sample of sliver was tested, but many sets of observations on several wires cut from the same coil gave very consistent results.

TABLE III.

Platinum. $T_0 = 273°$ K. Av. Pressure = 0.0014 Mm.			Platinum. $T_0 = 373°$ K. Av. Pressure = 0.003 Mm.		
Temp. in Degrees K.	Emission in Watts per Sq. Cm.	σ'.	Temp. in Degrees K.	Emission in Watts per Sq. Cm.	σ'.
647	0.3293	2.90×10^{-15}	642	0.2586	2.43×10^{-15}
698	0.4562	2.75×10^{-15}	699	0.3954	2.37×10^{-15}
772	0.7104	2.59×10^{-15}	725	0.4886	2.44×10^{-15}
834	0.9776	2.42×10^{-15}	802	0.7233	2.18×10^{-15}
879	1.2477	2.38×10^{-15}	870	1.0522	2.11×10^{-15}
914	1.4726	2.31×10^{-15}	917	1.3845	2.14×10^{-15}
981	2.0338	2.24×10^{-15}	944	1.6741	2.23×10^{-15}
991	2.1597	2.26×10^{-15}	979	2.0582	2.29×10^{-15}
1,018	2.4129	2.21×10^{-15}	1,007	2.3975	2.31×10^{-15}
1,046	2.7412	2.19×10^{-15}	1,014	2.8930	2.33×10^{-15}
1,091	3.3427	2.16×10^{-15}	1,048	2.9482	2.33×10^{-15}
1,118	3.7820	2.16×10^{-15}	1,100	3.7880	2.35×10^{-15}
1,148	4.2532	2.13×10^{-15}	1,135	4.4696	2.37×10^{-15}

TABLE IV.

Nichrome. $T_0 = 273°$ K. Av. Pressure = 0.0002 Mm.			Nickel. $T_0 = 273°$ K. Av. Pressure = 0.0002 Mm.		
Temp. in Degrees K.	Emission in Watts per Sq. Cm.	n.	Temp. in Degrees K.	Emission in Watts per Sq. Cm.	n.
325	0.0417	4.13	463	0.0258	4.65
392	0.1168	4.17	518	0.0462	4.66
483	0.3259	4.19	603	0.1000	4.67
540	0.5973	4.18	643	0.1343	4.70
673	0.9080	4.14	661	0.1526	4.67
823	1.2349	4.06	711	0.1935	4.68
1,013	1.8913	4.00	773	0.2408	4.62
1,073	2.6900	4.02	803	0.2808	4.62
1,113	3.6500	4.05	883	0.4498	4.63
1,163	5.3282	4.07	923	0.5852	4.63
1,178	5.9170	4.07	951	0.7012	4.64
1,228	8.2590	4.10	1,028	1.0484	4.64
1,258	9.8000	4.11	1,071	1.2553	4.64
1,283	11.5910	4.12	1,123	1.5499	4.64
1,308	13.3820	4.13	1,181	1.9324	4.64
			1,283	2.9572	4.65

Platinum.—Two specimens of platinum were tested with good agreement in results. Many sets of observations were taken on each specimen. The temperature-resistance curve for the specimen contained in this report was plotted from the equation

$$\rho_t = \rho_0(1 + \alpha t + \beta t^2),$$

where $\rho_0 = 1$ ohm, $\alpha = 0.0028$, $\beta = -0.00000033$. The enclosure was brass coated within with lamp-black. The diameter of the wire was 0.0159 cm. and its length between potential points was 40.3 cm. The emission was found to conform to the equation

$$E = c'T^5,$$

TABLE V.

Iron. $T_0 = 273°$ K. Av. Pressure = 0.0002 Mm.		
Temp. in Degrees K.	Emission in Watts per Sq. Cm.	c'.
696	0.2230	3.54×10^{-17}
753	0.3103	3.35×10^{-17}
805	0.4362	3.25×10^{-17}
853	0.5661	3.06×10^{-17}
915	0.7952	2.89×10^{-17}
975	1.1604	2.99×10^{-17}
1,083	2.1810	3.13×10^{-17}
1,133	2.7135	3.04×10^{-17}
1,180	3.3628	3.00×10^{-17}
1,192	3.5813	3.02×10^{-17}
1,233	4.7000	3.29×10^{-17}
1,240	4.9420	3.35×10^{-17}
1,303	7.8800	3.99×10^{-17}

TABLE VI.

Platinum.		Silver.		Nichrome.	
Temp. in Degrees K.	Resistance in Ohms.	Temp. in Degrees K.	Resistance in Ohms.	Temp. in Degrees K.	Resistance in Ohms.
273	1.000	273	1.000	273	1.000
373	1.280	323	1.145	317	1.020
473	1.553	373	1.350	339	1.033
573	1.820	423	1.510	398	1.045
673	2.079	477	1.720	448	1.058
773	2.332	523	1.889	503	1.073
873	2.579	574	2.070	556	1.088
973	2.816	623	2.275	606	1.103
1,073	3.049	683	2.520	668	1.118
1,173	3.275	723	2.685	735	1.128
1,273	3.494	792	3.000	867	1.133
1,373	3.706	823	3.105	945	1.138
1,473	3.911	854	3.240	1,018	1.143
1,573	4.109	913	3.545	1,058	1.149
1,673	4.300	953	3.730	1,103	1.154
		1,073	4.355	1,188	1.169
				1,238	1.179
				1,285	1.199
				1,320	1.219
				1,352	1.239
				1,375	1.249
				1,503	1.279

where c has the mean value of 2.36×10^{-15} when the enclosure was at 273° K. and 2.29×10^{-15} when it was at 373° K.

Nickel.—The diameter of the wire for which data are here given was 0.0455 cm. and its length between potential points was 29.8 cm. The energy-temperature cuvre suffers a slight variation in the neighborhood of the temperature at which recalescence occurs. In this neighborhood the emission increases less rapidly with rising temperature than either above or below. The variation in emitting power occurs at about the same temperature as that at which there is a pronounced variation in the coefficient of electrical resistance but does not persist over as great a range of temperature. The equation $E = c'T^n$ does not hold for nickel as well as for platinum. By holding c' constant — $c' = 1.04 \times 10^{-14}$— the variations in n are exhibited. The data for nickel are given in Table IV. n has the average value 4.648. Since nickel showed a

TABLE VII.

Iron.		Nickel.	
Temperature in Degrees K.	Resistance in Ohms.	Temperature in Degrees K.	Resistance in Ohms.
273	1.000	273	1.000
308	1.202	348	1.400
376	1.568	448	1.970
411	1.777	512	2.649
481	2.196	555	2.868
511	2.406	591	3.200
588	2.945	625	3.542
611	3.138	651	3.808
669	3.644	704	3.945
694	3.870	730	4.028
763	4.541	766	4.138
813	5.037	790	4.243
851	5.491	824	4.359
904	6.020	900	4.621
948	6.799	980	4.801
983	7.322	1,009	4.966
1,028	8.211	1,070	5.132
1,093	8.891	1,113	5.298
1,177	9.361	1,153	5.463
1,231	9.623	1,193	5.610
1,271	9.806	1,273	5.920
1,290	9.937		
1,328	10.198		
1,363	10.632		

variation from the Stefan equation at about the temperature 650° K. several different samples were tested with many sets of observations. The results were uniformly the same. With any given sample the curves could be reproduced with great accuracy.

Iron.—The diameter of the wire for which data are here given was 0.0476 cm., and its length between potential points was 29.9 cm. There is a pronounced change in the coefficient of electrical resistance in iron at about the temperature 1050° K. However, contrary to expectation, no variation in its emitting power was detected at this temperature. The energy-temperature curve was regular and conformed to Stefan's equation very well. This point was examined carefully. Different samples of iron were tested with great care but none of them showed any variation in emitting power at this temperature. c' has the average value 3.23×10^{-17} when n is taken equal to 5.55.

Nichrome.—The diameter of the wire was 0.046 cm. and its length between potential points was 24.6 cm. The temperature-resistance curve for nichrome suffers a pronounced change in the temperature interval 743° K. to 1043° K. In this same temperature range the energy-temperature curve varies. The emission from nichrome does not conform well to the equation $E = c'T^n$. The variations from this equation are shown by holding c' constant—$c' = 1.76 \times 10^{-12}$—and calculating n, Table IV. The samples of nichrome which were tested were all cut from the same coil. Many sets of observations were taken with uniformly consistent results.

This investigation was undertaken at the suggestion of Professor Augustus Trowbridge. The author takes this occasion to express his appreciation for the interest taken in this work by Professor Trowbridge and for the many helpful suggestions which were offered by him during its progress.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
June, 1914.

THE MAXWELL DISTRIBUTION LAW IN NEWTONIAN AND NON-NEWTONIAN MECHANICS.

BY GILBERT N. LEWIS AND ELLIOT Q. ADAMS.

SINCE Maxwell first obtained the law for the distribution of the molecules of a perfect gas with respect to velocity, numerous derivations of this law have been offered, and much discussion has ensued as to the logical rigor of these derivations. The method which has received the most attention in recent years is based upon the theorem of Liouville, and it is this method which has been used by Jüttner[1] and more recently by Tolman[2] in discussing what modifications of the Maxwell equation are necessitated by the assumption of the mechanics of relativity.

Without discussing the difficult questions which may arise, concerning the complete mathematical validity of these derivations, we wish, on account of the great theoretical importance of this subject, to offer an entirely independent method of deriving the distribution law,—in the non-Newtonian as well as in the Newtonian case,—a method which, as far as we can now see, is as rigorous as the fundamental principles of mechanics and geometry on which it is based.

In order to be as free as possible from non-essentials we shall consider the simplest possible case, namely, an ideal monatomic gas, in other words a system of like particles which are capable of acquiring only kinetic energy, situated in no directing field such as that of gravitation, and subject to the restriction that no portion of the gas, macroscopically considered, be in motion with respect to the common center of mass of the system. Our fundamental assumption will therefore be one of complete symmetry with respect both to position and to direction.

THE NEWTONIAN CASE.

Let us consider a system of gas molecules in temperature equilibrium. At a given instant, the vector momentum of each particle being determined, we may plot from a common center in three-dimensional space each of these vectors. The points which are the termini of these vectors

[1] Ann. Phys., 4, 34, 856 (1911).

[2] Phil. Mag., 28, 583 (1914).

will therefore be used to represent the momenta of the several particles, both in magnitude and direction. Moreover, since the particles are of the same size, these points may also represent the individual velocities. Thus two points in Fig. 1, lying on the same radius vector from O, represent two particles moving in the same direction with different velocities; while two points equidistant from O represent two particles moving in different directions with the same scalar velocity.

Now since by assumption the particles can acquire no energy except kinetic, every collision between two particles[1] is elastic,[2] and the relative velocity of the two will change in direction but not in magnitude. Since the center of gravity of the two particles taken together remains fixed, a pair of points (A, B) must by collision be converted into two other points (C, D) such that the center of the line CD coincides with the center of the line AB. In other words, CD is another diameter of the sphere of which AB is a diameter.

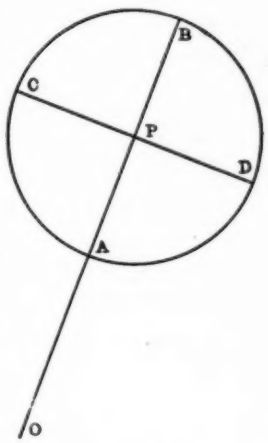

Fig. 1.

If, in the figure, O represent the point of rest and P the center of any sphere, of which AB and CD represent any two diameters, the chance that a collision of a particle at A with one at B give a particle at C and one at D is equal to the chance that a collision of a particle at C with one at D produce particles at A and B, and this regardless of the nature of the forces during collision, for to an observer at P the diameters AB and CD differ only in direction. This is equivalent to the statement that the fraction of collisions (A, B) resulting in (C, D) and the fraction of collisions (C, D) resulting in (A, B) are equal.

If the whole space of our figure be divided into equal infinitesimal regions the condition of equilibrium with respect to velocity distribution is that the number of particles leaving any region A in a given time is the same as the number entering. Owing to our conditions of symmetry this is equivalent to the more special condition that the number of

[1] By the definition of an ideal gas we may exclude the case of simultaneous collisions of three or more particles, for by diminishing the concentration of the gas the number of such collisions falls off with the cube, at least, of the concentration while the number of simple collisions falls off only with the square.

[2] In order that the particles should possess no energy except kinetic, collision must be not only elastic but instantaneous, that is, the time during which the particles are subjected to mutual forces must be negligible in comparison with the interval between collisions. This is a condition approached by an actual monatomic gas as the concentration is indefinitely diminished.

particles in the region A which collide with particles in any other region B to give particles in regions C and D (where CD is another diameter of the sphere AB) is equal to the number of particles from C and D going into A and B. If this be true throughout the space the number of particles in each region will obviously remain constant.

We have shown that the chance that any pair of particles leaving the regions A and B enter the regions C and D is equal to the converse chance. The number of collisions of particles in A with particles in B to produce particles in C and D will be proportional to the product of the number in A by the number in B, and the number of collisions between particles in C and D to give particles in A and B is proportional to the product of their numbers. Moreover the proportionality factor will be the same in both cases owing to the symmetry about the point P which we have already pointed out. If n_A represent the number of particles in A, etc., our condition of thermal equilibrium is therefore

$$n_C n_D = n_A n_B. \tag{1}$$

Since the distribution of particles with respect to velocity depends only upon the scalar magnitude of the velocity, that is, upon the scalar distances OA, etc., n is a function of that scalar distance only, and we now have sufficient data for the complete determination of this functional relation.

Since equation (1) is true in general, it is true for the special case which we shall now, for convenience, assume, namely when AB is in the line of OP, and CD is perpendicular to it. In this case $OC = OD = \sqrt{r^2 + a^2}$ where $OP = r$ and $AP = a$. Also $OA = r - a$, $OB = r + a$, and, writing $f(OA)$ for n_A, equation (1) becomes

$$[f(\sqrt{r^2 + a^2})]^2 = f(r - a)f(r + a).$$

Taking logarithms

$$2 \ln f(\sqrt{r^2 + a^2}) = \ln f(r - a) + \ln f(r + a).$$

If now we write $\ln f(\sqrt{x}) = F(x)$

$$2F(r^2 + a^2) = F([r - a]^2) + F([r + a]^2).$$

If $(r - a)^2 = x$ and $(r + a)^2 = y$, $r^2 + a^2 = (x + y)/2$, and

$$2F\left(\frac{x + y}{2}\right) = F(x) + F(y).$$

Taking the partial derivative with respect to x, and to y

$$F'\left(\frac{x + y}{2}\right) = F'(x),$$

$$F'\left(\frac{x + y}{2}\right) = F'(y),$$

where F', as usual, represents the derivative of F with respect to its own argument.

Since we have already used the fact that x and y are completely independent, OA and OB being any two distances, F' is a constant and F a linear function, thus, choosing r^2 as the argument,

$$F(r^2) = A + Br^2,$$

or by our previous convention,

$$ln\, f(r) = A + Br^2,$$

$$f(r) = e^{A+Br^2} = ke^{Br^2},$$

where $k = e^A$.

Now $f(r)$ is the number of particles in a certain infinitesimal volume of our space, and r measures the numerical velocity. In order to give the number of particles between r and $r + dr$, or rather between the velocities v and $v + dv$, we must write

$$\frac{dn}{dv} = \alpha v^2 e^{Bv^2}, \tag{2}$$

where the relation of the new constants to the former depends upon the units of r and v. This is the customary form of the Maxwell equation.

THE GENERAL DISTRIBUTION LAW.

In generalizing the proof which we have just given, to include the concepts of relativity and thus to obtain an equation valid for particles of any mass at any temperature, we must proceed with great care. Thus, in the preceding paragraphs, we have assumed, as is justified in the limiting Newtonian case, that there is no ambiguity in speaking of the number of particles leaving a certain momentum region in a given time; whereas in the kinematics of Einstein, time has no definite significance except with reference to an arbitrary zero of velocity. But, if care be taken to eliminate such ambiguities, the first postulate of relativity itself permits the use of just such principles of symmetry as we have already employed.

Since the epoch-making work of Minkowski the science of kinematics may be regarded as a pure four-dimensional geometry. If intervals of space and time are to be made real intervals of length in this geometry, the geometry proves to be the same as the Euclidean except for a single postulate, namely, the Euclidean postulate that in any plane a circle may be constructed by the rotation of any line about one of its extremities. The non-Euclidean geometry which results from a change in this postu-

[1] "The Space-Time Manifold of Relativity. The Non-Euclidean Geometry of Mechanics and Electromagnetics," Proc. Amer. Acad., 48, 389 (1912).

late has been discussed in some detail by Wilson and Lewis,[1] whose nomenclature and methods we shall employ here.

According to the principle of relativity, the choice of a zero velocity and thus of a standard of time, is as purely arbitrary as the choice of special coördinates in ordinary geometry. But in a given problem such a choice is often prescribed by considerations of convenience, and this is the case in the problem we are now considering. We shall assume the system of gas molecules as a whole to be at rest, and the locus in four dimensions of the center of mass of the system to determine the standard axis of time. This standard of time determines standard space and standard volume. We may, however, when occasion arises, consider any one of the molecules at rest and thus fix what we may call the proper time (Eigenzeit), and the proper volume, corresponding.

In order to give a geometrical picture of the system under consideration we may in the first instance consider a system of molecules which are free to move not in a given volume but in a given plane area, like balls upon a billiard table. We thus reduce the geometry from four dimensions to three, and we have attempted in Fig. 2 to illustrate this simpler case. But in order to avoid a repetition of the proof we may

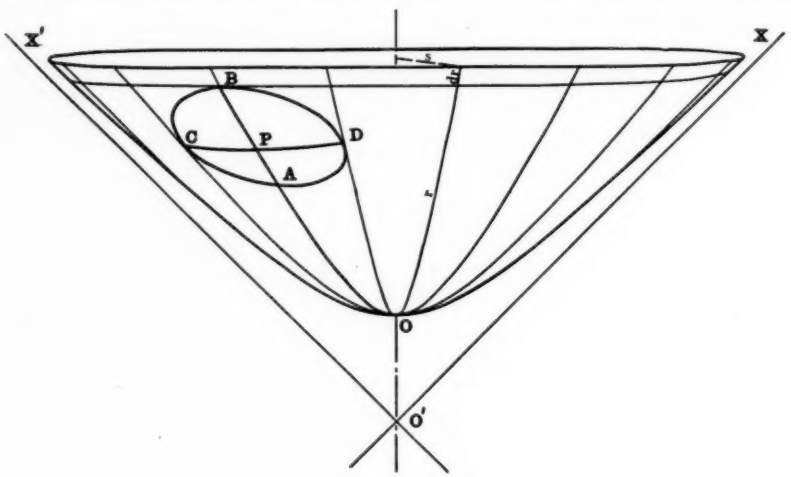

Fig. 2.

generalize as we proceed, substituting spheres for circles, pseudo-spheres for pseudo-circles, hypercones for cones, etc.

In the figure, $O'X$, $O'X'$ are elements of the singular cone (hypercone). They represent for example the locus of something moving with the velocity of light. A line within the cone (hypercone) represents the locus of any particle moving with a velocity less than that of light. If the

particle is unaccelerated this locus is straight, and a vector along this line, of magnitude m_0—the mass of the particle at rest—is the "vector of extended momentum" of the particle. Its projections on a chosen time axis and upon the perpendicular plane (planoid) are respectively the ordinary mass and momentum of the particle.

If $O'O$ represent such a vector, then the vectors representing all possible motions of the same particle, or of other particles of the same m_0, terminate on the pseudo-sphere (hyper-pseudo-sphere) shown in the figure. For convenience we may choose such units that the m_0 of the particles and therefore the radius of the pseudo-sphere (hyper-pseudo-sphere) is unity. If now the vectors of extended momentum of all the gas molecules be drawn from O', the terminus of each will be a point upon the pseudo-sphere (hyper-pseudo-sphere) and our problem is the determination of the distribution of these points upon that surface (hyper-surface).

We shall show in an appendix to this article that this surface (hyper-surface) has all the properties of a Lobatschewskian plane (space) of unit space-constant, and the straight line of which is its intersection with any plane through O'. Our problem is therefore analogous to the one we have already treated, but the geometrical laws are those of Lobatschewskian space.

In the first place it is evident that if $O'O$ is the locus of one of the particles at rest, the distribution of the points representing vector-termini must be symmetrical about O; that is, if the whole surface (space) be divided as before into equal infinitesimal areas (volumes) the number in the region at a point A is a function of the distance OA alone.

If A and B be any two points, a collision between a particle at A and a particle at B will result in particles at some points C and D such that CD and AB are diameters of the same circle (sphere). This result follows at once from the law of the conservation of extended momentum,[1] or we may obtain it without the aid of this principle by considering an observer moving with a particle at P, the middle point of AB. To such an observer particles A and B are moving with equal and opposite velocities and their common center of mass is at rest. The center of mass will evidently remain at rest after collision and if the collision is elastic the scalar velocities with respect to that center will also remain unchanged. In other words, C and D must be such points that P is the center of CD and that $PC = PA$.

Now it is again evident from the symmetry about P that a collision between a pair of particles (A, B) has the same chance in resulting in a

[1] Wilson and Lewis, loc. cit.

pair of particles (C, D) as a collision of the pair (C, D) has to produce the pair (A, B). Therefore the condition for permanency of distribution is that, from the standpoint of any one observer, the number of particles passing from (A, B) to (C, D) per second is equal to the number passing from (C, D) to (A, B). We shall naturally choose the common center P as the point of observation.

Now as far as the individual molecules are concerned the symmetry about P is perfect and we might be tempted to assume that once more the condition of stability could be written $n_C n_D = n_A n_B$. But we must remember that the particles A, B, C and D are not symmetrically placed with respect to the center of mass of the system, or, in other words, the enclosure containing the gas is not at rest with respect to P.

It was not necessary to point out in our previous proof that the frequency of collisions depends not merely upon the number of molecules of the kind considered but also upon the volume of the enclosure, for this volume was assumed to have definite significance, without further qualification. But in our present case the volume of the enclosure will appear different to observers moving with the different molecules. If, however, we consider not the number but the concentration by volume of particles in a region A, and if by concentration we mean the "proper" concentration as measured by an observer at A, then we may employ the principle of symmetry and write as the condition of equilibrium

$$C_C C_D = C_A C_B, \tag{3}$$

where C_A is the proper concentration at A, measured at A, C_B the proper concentration at B, measured at B, etc.

We shall once more choose the simple case where AB is an extension of OA and CD is perpendicular. Replacing C_A by $f(OA)$ we may write

$$f(OC)f(OD) = f(OA)f(OB)$$

and according to the proposition corresponding to the Pythagorean in Lobatschewskian geometry

$$\cosh(OC) = \cosh(OD) = \cosh(OP)\cosh(PC),$$

or writing, as before, $OP = r$, $PA = PC = a$, $OA = r - a$,

$$OB = r + a,\ OC = \cosh^{-1}(\cosh r \cosh a):$$

$$[f(\cosh^{-1}[\cosh r \cosh a])]^2 = f(r - a)f(r + a).$$

This functional equation may be uniquely solved as follows: Taking logarithms

$$2\ ln\ f(\cosh^{-1}[\cosh r \cosh a]) = ln\ f(r - a) + ln\ f(r + a).$$

Defining a new function,

$$F(x) = ln\ f(\cosh^{-1} x), \tag{4}$$

$$2F(\cosh r \cosh a) = F(\cosh[r - a]) + F(\cosh[r + a]),$$

or since $\cosh r \cosh a = \frac{1}{2}\cosh(r-a) + \frac{1}{2}\cosh(r+a)$,

$$2F(\tfrac{1}{2}\cosh[r-a] + \tfrac{1}{2}\cosh[r+a]) = F(\cosh[r-a]) + F(\cosh[r+a]).$$

Taking partial derivatives with respect to $\cosh(r-a)$ and $\cosh(r+a)$, respectively, which are independent since $(r-a)$ and $(r+a)$ may be varied independently at will,

$$F'(\tfrac{1}{2}\cosh[r-a] + \tfrac{1}{2}\cosh[r+a]) = F'(\cosh[r-a]),$$

$$F'(\tfrac{1}{2}\cosh[r-a] + \tfrac{1}{2}\cosh[r+a]) = F'(\cosh[r+a]),$$

or $F'(\cosh[r-a]) = F'(\cosh[r+a])$, hence F' is evidently a constant and F a linear function, or,

$$F(\cosh r) = A + B\cosh r,$$

whence by (4)

$$ln\, f(r) = A + B\cosh r$$

and

$$f(r) = ke^{B\cosh r} \tag{5}$$

where $k = e^A$.

Now $f(r)$ is the proper concentration at the distance r from O. If we take unit volume of the gas as measured by an observer at rest with respect to the system as a whole, that which is the proper unit volume at the distance r is less than unity measured in standard volume, in the ratio $\operatorname{sech} r$. Hence the standard density is greater than the proper density at that point in the ratio $\cosh r$, for if r is the distance from O to any point such as A, then the angle $OO'A$ is equal to this arc r of the pseudo-circle, and the hyperbolic tangent of this angle ($\tanh r$) is the ordinary scalar velocity in the space perpendicular to OO' (the velocity of light taken as unity). Therefore $\cosh r = 1/\sqrt{1-v^2}$ which is the familiar factor for the reduction of volumes at rest to volumes in motion.

The actual number of particles in any one of our equal infinitesimal regions is therefore not $f(r)$, but $\cosh r\, f(r)$. We shall therefore write for this number from equation (5)

$$\cosh r\, f(r) = k\cosh r\, e^{B\cosh r}.$$

Now this number is an infinitesimal region at distance r, multiplied by the number of such regions lying between r and $r+dr$, which is proportional to the area (volume) of the space thus cut out, gives the total number of particles lying between these limits. This area (volume) is equal to dr multiplied by the circumference (surface) of the Euclidean circle (sphere) of radius s, namely, $2\pi s\, dr$, $(4\pi s^2 dr)$. Now s in our relativity geometry is equal to $\sinh(OO'A) = \sinh r$, and (abandoning now the case of particles in a plane)

$$\frac{dN}{dr} = 4\pi k \sinh^2 r \cosh r\, e^{B\cosh r}. \tag{6}$$

This is the fundamental law for the distribution of gas particles with respect not to velocity but to the angle in our relativity space, of which the hyperbolic tangent is the velocity. We may put it in another form, writing $\cosh r - 1 = \text{versh } r$; $4\pi k e^B = A$,

$$\frac{dN}{dr} = A \sinh^2 r \cosh r \, e^{B \text{ versh } r}. \tag{7}$$

In the limiting case of small velocities, $r = \sinh r = \tanh r = v$, and the equation gives the simple Maxwell law already obtained where $A = \alpha$ and $B = -2\beta$. If we wish to get the general distribution law with respect to velocity, which, however, has no fundamental importance, we may write, since $d \tanh r = \text{sech}^2 r \, dr$,

$$\frac{dN}{dv} = A \sinh^2 r \cosh^3 r \, e^{B \text{ versh } r}$$

$$= A \frac{v^2}{(1 - v^2)^{\frac{5}{2}}} e^{B\left(\frac{1}{\sqrt{1-v^2}} - 1\right)}. \tag{8}$$

Similarly it is obvious that we may obtain the distribution of N with respect to any of the other hyperbolic functions of r, and two of these have important physical significance, for $m_0 \sinh r = M$, the momentum of a particle; and $m_0 \cosh r = m$, its mass (or total energy); while $m_0 \text{ versh } r = E$, the additional mass due to its motion (or its kinetic energy). Thus

$$\frac{dN}{d \sinh r} = A \sinh^2 r \, e^{B(\sqrt{1+\sinh^2 r} - 1)}, \tag{9}$$

$$\frac{dN}{dM} = \frac{A}{m_0^3} M^2 e^{B\left(\sqrt{1+\left(\frac{M}{m_0}\right)^2} - 1\right)}, \tag{10}$$

$$\frac{dN}{d \cosh r} = A\sqrt{\cosh^2 r - 1} \cosh r \, e^{B(\cosh r - 1)}, \tag{11}$$

$$\frac{dN}{dm} = \frac{A}{m_0^3}\sqrt{m^2 - m_0^2} \, m \, e^{B\left(\frac{m}{m_0} - 1\right)}, \tag{12}$$

$$\frac{dN}{d \text{ versh } r} = A\sqrt{2 \text{ versh } r - \text{versh}^2 r}(1 + \text{versh } r) \, e^{B \text{ versh } r}, \tag{13}$$

$$\frac{dN}{dE} = \frac{A}{m_0^3}\sqrt{2m_0E + E^2} \, (m_0 + E) \, e^{\frac{BE}{m_0}} \tag{14}$$

Equation (10) shows the distribution of particles with respect to momentum, equation (12) with respect to mass or total energy, and equation (14) with respect to kinetic energy. These equations are in

complete accord with the distribution equation obtained in a very different way by Jüttner.[1]

If we integrate any one of these equations over the entire range we may write the result equal to the total number of particles. Similarly an equation for the total kinetic energy can be obtained, and the constants A and B are thus determined in terms of those quantities. As shown by Jüttner, B can be very simply expressed by the equation

$$B = -\frac{m_0}{kT}, \tag{15}$$

while A has as yet been obtained only in terms of definite integrals which may be reduced to Bessel functions.

Tolman has already pointed out that in the case of a gas composed of particles of zero (infinitesimal) mass at rest the distribution equation assumes a simple, integrable form. Molecules of zero m_0 may have no physical significance, it is therefore important to note that the equations assume the same simple form for any molecules at very high temperatures. Thus equation (14), when m_0 is infinitesimal, or when T approaches infinity, assumes the form

$$\frac{dN}{dE} = \frac{A}{m_0^3} E^2 e^{-\frac{E}{KT}}.$$

This same equation results likewise from equations (10) and (12) for when $v = 1$, $E = M = m$. The constant A/m_0^3 is easily evaluated, since integration from zero to infinity gives $N = 2k^3T^3A/m_0^3$, whence

$$\frac{dN}{dE} = \frac{N}{2k^3T^3} E^2 e^{-\frac{E}{KT}}. \tag{16}$$

The total energy of particles whose individual energies lie between E and $E + dE$ is therefore

$$dU = EdN = \frac{N}{2k^3T^3} E^3 e^{-\frac{E}{KT}} dE. \tag{17}$$

The equations have been developed in this form for reasons which will be brought out in another place.

APPENDIX.

In order to prove that all the propositions of a certain geometry are applicable to a given spatial system, it suffices to show that the postulates of the geometry are valid for that space. The postulates of Lobat-

[1] There is a slight confusion in Jüttner's paper owing to his designation of $m_0/\sqrt{1-v^2}$, which is the total mass or energy, as "lebendige Kraft," but this does not affect seriously the important conclusions which he obtains.

schewskian geometry are known to be identical with those of Euclidean geometry except that the parallel postulate is replaced by the following:

Through any point not on a given straight line two and only two parallel (asymptotic) straight lines can be drawn.

From our knowledge of the non-Euclidean geometry of relativity we may readily show the validity of the Lobatschewskian postulates upon the surface of the pseudo-sphere, the intersections of which with planes drawn through the center of the pseudo-sphere are the "straight lines" of the space.

Since any two points on the surface of the pseudo-sphere, together with the center, determine one and only one plane, one and only one straight line can be drawn through any two points.

Since rotation about a line joining the center with any point on the surface leaves that point fixed and carries every other point through a closed path which is completely in the surface, and of which the radii in that surface are mutually congruent, the system obeys the Euclidean postulate of rotation.

It is evident from the construction of the surface that two straight

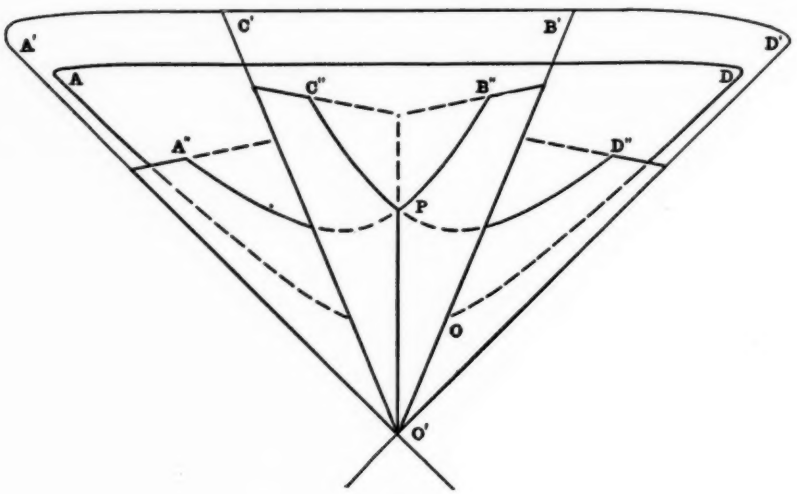

Fig. 3.

lines are parallel (asymptotic) when and only when the two planes which determine them intersect in an element of the singular cone. The plane which determines a given line cuts the cone in two elements, each of which determines, together with any point outside the given line, a plane which intersects the surface in a line asymptotic to the given line. Thus in Fig. 3 the plane $A'O'D'$ determines the line AOD. P represents

a point off this line and determines with $O'A'$ the plane $A'O'B'$ and hence the line $A''PB''$; likewise P determines with $O'D'$ the plane $C'O'D'$ and hence the line $C''PD''$. PA'' and OA are asymptotic to each other and to $O'A'$, and PD'' and OD are asymptotic to each other and to $O'D'$.

Such other minor postulates as are necessary to define completely the geometry, such as the infinite extension of the straight line, also follow obviously from the construction. Moreover, owing to the known identity of the Lobatschewskian with the log.-spherical geometry of Taurinus, it is evident that the constant of the space determined by a pseudo-sphere of unit radius is unity.

When we consider the hyper-pseudo-sphere with its three dimensional boundary we see that every intersection of this boundary with a planoid drawn through its center is a pseudo-sphere of unit radius. If we regard these intersections as Lobatschewskian "planes," then since any two intersecting lines determine a Lobatschewskian plane the space is Lobatschewskian.

This is not the place to dwell further upon the various interesting features of the geometrical constructions in this non-Euclidean geometry of relativity, but we believe that the one which we have just employed gives a far better picture of the properties of Lobatschewskian space than can be obtained by the aid of the figure constructed in Euclidean space which is also known as the pseudo-sphere. Thus, for example, one of the interesting figures in Lobatschewskian space, the horocycle (horo-sphere), is in our construction merely the intersection of the pseudo-sphere (hyper-pseudo-sphere) with a plane (planoid) parallel to a tangent plane (planoid) of the singular cone (hypercone). The surface of the other pseudo-sphere (of one sheet) which envelopes the singular cone also presents a most interesting geometry which has, we believe, not been discussed hitherto.

CHEMICAL LABORATORY OF THE UNIVERSITY OF CALIFORNIA,
January 9, 1914.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

NEW YORK MEETING, FEBRUARY 27, 1915.

The seventy-sixth regular meeting of the Physical Society was held in Fayerweather Hall, Columbia University, New York, on Saturday, February 27, at 10:00 o'clock. The following papers were presented:

The Joule Free Expansion Experiment and the Correction of The Constant Volume Gas Thermometer. LYNDE P. WHEELER.

The Forms of Physical Equations. (By title.) E. BUCKINGHAM.

Electrical Characteristics of X-Ray Tubes. J. S. SHEARER and P. T. WEEKS.

Fluorescence and Absorption of Certain Organic Substances. R. C. GIBBS and K. GIBSON.

Absorption of Certain Glasses of Known Composition. K. GIBSON.

Fluorescence of Frozen Solutions of Uranyl Salts. (By title.) W. HOWES.

Fluorescence and Absorption of Uranyl Ammonium Chloride. EDWARD L. NICHOLS.

Response of a Silicon Detector to Short Electric Waves. ERNEST MERRITT.

Agglomeration Theory of Variation of Specific Heat of Solids with Temperature. A. H. COMPTON.

Construction of Crystals with Theoretical Atom. A. C. CREHORE.

Notes on the Atomic Nuclei. WILLIAM DUANE.

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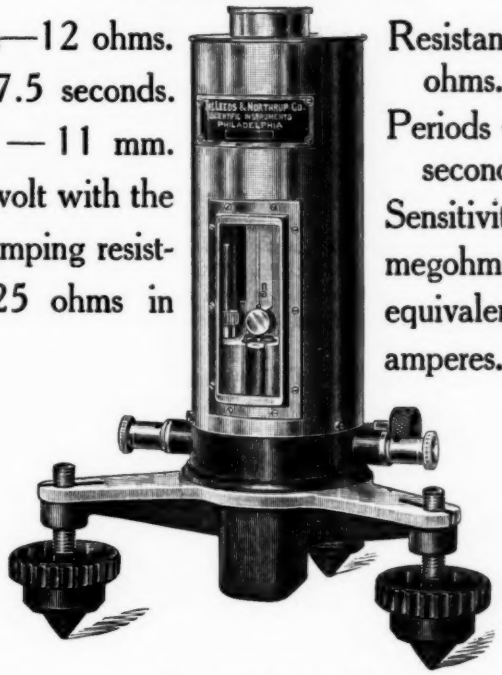

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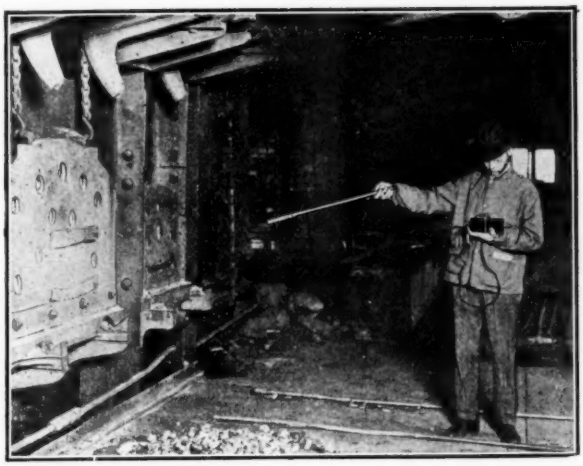

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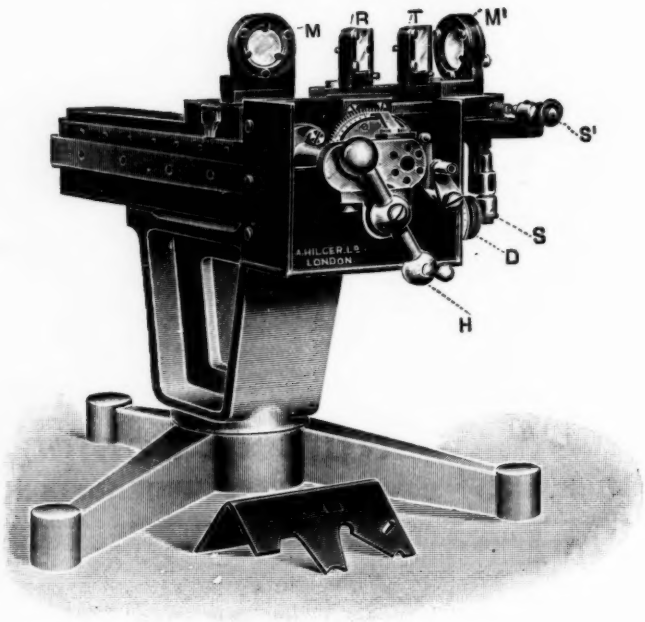

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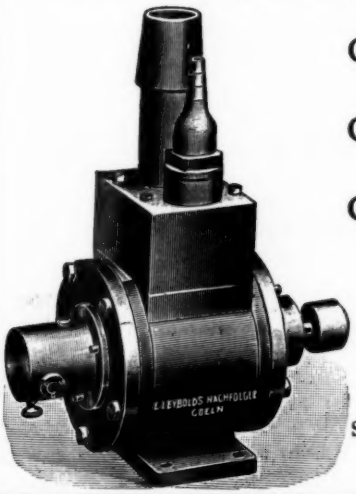

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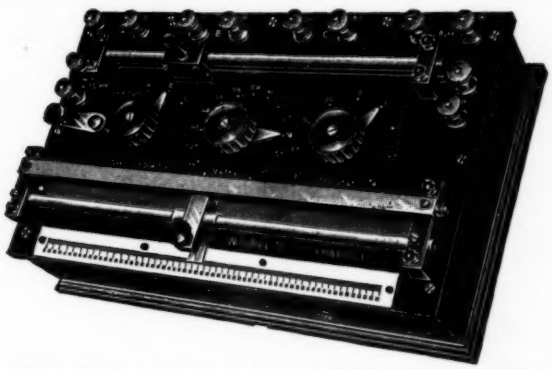

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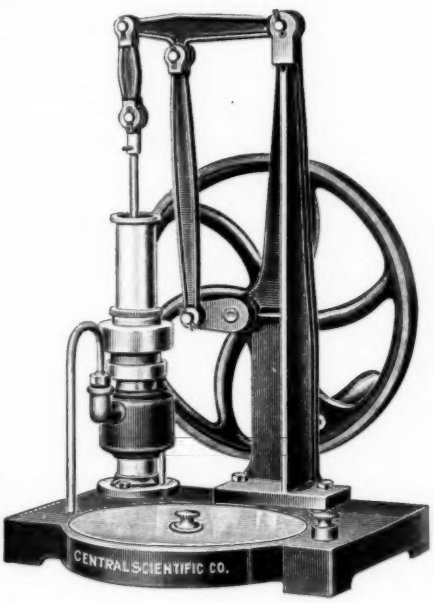

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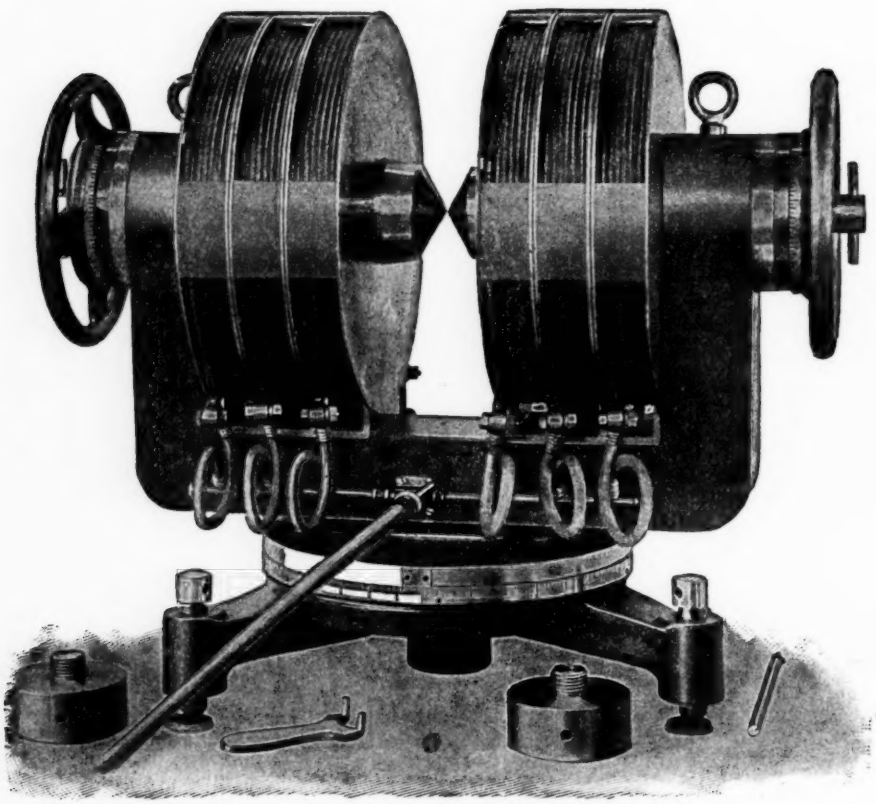

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35 "	3 "	29630 "

Size of the air-gap		Fields
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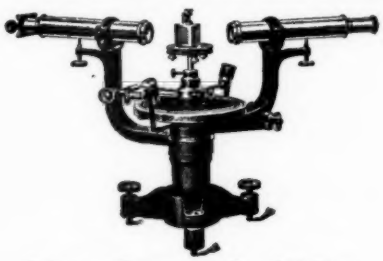

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